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General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1	$H_0: \mu = 768$ $H_1: \mu \neq 768$	B1		(Both)
	Test statistic: $z = \dfrac{764.8 - 768}{8/\sqrt{18}}$	M1		
	$= -1.70$	A1		(-1.697)
	$z_{crit} = \pm 1.96$	B1		($z_{crit} = 1.96$ or $z_{crit} = -1.96$)
	$\Rightarrow$ Accept H_0	A1		
	No evidence at the 5% level of significance, to deny Yvonne's claim.	E1	6	
	Total		**6**	
2(a)(i)	$X \sim \text{Po}(5.0)$ $\Rightarrow \text{P}(X < 4) = \text{P}(X \leq 3)$			$(0.440$ to $0.441)$ for B1
	$= 0.265$	B2	2	CAO
(ii)	$Y \sim \text{Po}(1.5)$		2	
	$\Rightarrow \text{P}(Y = 4) = \dfrac{e^{-1.5} \times (1.5)^4}{4!}$	M1		
	$= 0.0471$	A1	2	(0.047 to 0.0471)
2(b)(i)	$T = X + Y \sim \text{Po}(6.5)$	B1		
	$\Rightarrow \text{P}(T > 5) = 1 - \text{P}(T \leq 5)$	B1		$(1 - 0.2237)$ **or** $(1 - 0.5265)$
	$= 1 - 0.369$ $= 0.631$	B1	3	
(ii)	$p = {}^8C_7 (0.631)^7 (0.369) +$ $(0.631)^8$	M1ft		ft on their p from (b)(i) Either part attempted
	$p = 0.11758 + 0.02513$	A1ft		(both parts correct)
	$= 0.143$	A1	3	AWFW 1.142 to 0.143 (CAO)
(c)(i)	Mean = 8	B1		CAO
	Variance = $s^2 = 16.9$ $(\text{sample variance} = 15.2)$	B1	2	(AWRT)
(ii)	Poisson not a good model for data	B1dep		
	Mean $\neq$ Variance	B1	2	
	Total		**14**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3	H_0 **:** no association between age and attitude to school reorganisation	B1		
	H_1 **:** association between age and attitude to school reorganisation			
		M1		E's attempted
		A1		correctly (at least 6 E's)

Age	Against	
	O_i	E_i
16 - 17	9	$6\frac{17}{65}$
18 - 21	17	$15\frac{24}{65}$
22 - 49	115	$116\frac{9}{13}$
50 - 65	41	$42\frac{9}{13}$
> 65	3	$3\frac{64}{65}$
Total	185	185

E_i
6.262
15.369
116.692
42.692
3.985

Age	Not Against	
	O_i	E_i
16 - 17	2	$4\frac{48}{65}$
18 - 21	10	$11\frac{41}{65}$
22 - 49	90	$88\frac{4}{13}$
50 - 65	34	$32\frac{4}{13}$
> 65	4	$3\frac{1}{65}$
Total	140	140

E_i
4.738
11.631
88.308
32.308
3.015

Row totals: $\widehat{11,27}$ 205, $\widehat{75.7}$ (325)

Column totals: 185, 140 (325) — B1 — Totals correct

$E_i's < 5$

$\therefore$ combine cells 16 – 17 and 18 –21 **also** 50 – 65 and 'over 65' to give: — M1 Attempt at combining rows; A1 Correctly

O_i	E_i	$\alpha = O_i - E_i$	α^2 / E_i
26	21.63	4.369	0.8825
115	116.69	−1.692	0.0245
44	46.68	–2.677	0.1535
12	16.37	–4.369	1.1662
90	88.31	1.692	0.0324
38	35.32	2.677	0.2029
325	325		2.462

ml — Final column attempted (dep M1)

$X^2 = 2.462$ — A1 — 2.4 to 2.5

$\nu = 2$ — B1

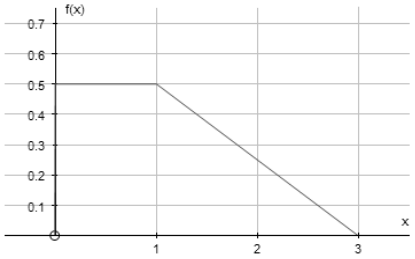
$\chi^2_{\nu=2}(0.95) = 5.991$ — B1ft — On their ν

Accept H_0 — A1ft

No real evidence at 5% level of significance to suggest any association between age and attitude to school reorganisation. — E1ft — 12 — (context)

	Total		**12**	

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	Sketch:  	B3	3	1 for straight line $0 \le x \le 1$ from (0, 0.5) to (1, 0.5) 1 for straight line $1 \le x \le 3$ from (1, 0.5) to (3, 0) 1 for axes [must have at least (0,0.5) (1,0) and (3,0) labelled]
(b)	$\mathrm{P}(X \le \eta) = \mathrm{F}(\eta) = 0.5$ $(\Rightarrow \quad \eta = 1$ (from graph))	M1 A1	2	AG
(c)	$\mu = \mathrm{E}(X) = \int_0^1 \left(\frac{x}{2}\right) \mathrm{d}x + \int_1^3 x\left(\frac{3-x}{4}\right) \mathrm{d}x$ $= \left[\frac{x^2}{4}\right]_0^1 + \frac{1}{4}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_1^3$ $= \frac{1}{4} + \frac{1}{4}\left[\left(\frac{27}{2} - 9\right) - \left(\frac{3}{2} - \frac{1}{3}\right)\right]$ $= \frac{1}{4} + \frac{5}{6} \quad (0.25 + 0.83\dot{3})$ $= 1\frac{1}{12}$	M1 A1 ml A1	4	Both integrals stated Either Correct limits used on both integrals +combined dep M1 (CAO)
(d)	Area of $\triangle$ $= \mathrm{P}\left(X > 2\frac{1}{4}\right) = \frac{1}{2} \times \frac{3}{4} \times \frac{3 - 2\frac{1}{4}}{4}$ $= \frac{3}{32} \times \frac{3}{4} = \frac{9}{128}$ $\therefore \quad \mathrm{P}\left(X < 2\frac{1}{4}\right) = 1 - \frac{9}{128}$ $= \frac{119}{128}(0.9296875)$	M1ft M1ft A1	3	**Alternative:** For $1 \le x \le 3$ $\mathrm{F}(x) = 1 - \frac{1}{8}(3-x)^2$ M1ft $\Downarrow$ $\mathrm{F}\left(2\frac{1}{4}\right) = 1 - \frac{1}{8} \times \frac{9}{16}$ M1ft $= \frac{119}{128}$ CAO

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(d)	**or**			**Alternative**
	$\int_{2\frac{1}{4}}^{3} \frac{3-x}{4}\,dx \left(= \frac{9}{128}\right)$ M1 ft			$f\left(2\frac{1}{4}\right) = \frac{3}{16} = 0.1875$
	$= 1 \int_{2\frac{1}{4}}^{3} \frac{3-x}{4} dx$ M1 ft			$P\left(X < 3\mu - \eta\right) = P\left(X < 2\frac{1}{4}\right)$
	$= 1 - \frac{1}{4}\left[3x - \frac{x^2}{2}\right]_{2\frac{1}{4}}^{3}$			$= \frac{1}{2} + \boxed{\frac{1}{2}\left(\frac{3}{16} + \frac{1}{2}\right) \times 1\frac{1}{4}}$ M1ft
	$= 1 - \frac{1}{4}\left[9 - \frac{9}{2} - \frac{27}{4} + \frac{81}{32}\right]$			$= \frac{1}{2} + \frac{55}{128}(0.4296875)$ M1ft
	$= 1 - \frac{1}{4} \times \frac{9}{32} = \frac{119}{128}$ A1			$= \frac{119}{128}$ (0.930) A1
	or $(1 - 0.0703125 = 0.9296875)$			
	Total		**12**	
5(a)(i)	$P(GG \text{ or } YY \text{ or } RR)$	M1		
	$= \frac{2}{10} \times \frac{1}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{4}{10} \times \frac{3}{9}$			
	$= \frac{2}{9}$	A1	2	(AG)
(ii)	$P\left(B\bar{B} \text{ or } \bar{B}B\right) = \frac{1}{10} \times \frac{9}{9} + \frac{9}{10} \times \frac{1}{9}$	M1		$\frac{1}{10} + \frac{9}{10} \times \frac{1}{9}$
	$= \frac{1}{5}$	A1	2	(AG)
(b)(i)				

	Same	1 Blue	Neither
x	135	145	−45
$P(X = x)$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$

B1

B1 2

(ii)	$E(X) = 135 \times \frac{2}{9} + 145 \times \frac{1}{5} + (-45) \times \frac{26}{45}$	M1		Multiply two rows of their table from (b)(i)
	$= 29 + 30 - 26$			
	$= 33$ pence	A1	2	AG
(c)(i)	$E(Y) = 104 - 3E(X)$	M1		
	$= 104 - 3 \times 33$			
	$= 5$ pence	A1		
	∴ Joanne would expect to win £5	A1	3	OE (eg 500p)

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(c)(ii)	$\mathrm{E}(X^2) = 9425$	B1		$(4205 + 4050 + 1170)$
	$\mathrm{Var}(X) = 9425 - 33^2 = \mathbf{8336}$			$sd(X) = 91.30$
	$\mathrm{Var}(Y) = 9 \times \mathrm{Var}(X)$	B1		
	$= 9 \times 8336$			$9 \times (\text{their Var}(X) > 0)$
	$= 75024$	M1		**or** $3 \times (\text{their sd}(X))$
	$\Rightarrow$ standard deviation (Y) = 274 pence	A1	4	273.9p or £2.74
	Total		**15**	
6(a)(i)	$\bar{x} = 43.5$	B1		
	$s = 2 \ (s^2 = 4)$	B1		
	Assumption: Weights of boxes are normally distributed	B1		
	$t_{0.975} = 2.365$	B1		
	95% CI for μ : $43.5 \pm 2.365 \times \frac{2}{\sqrt{8}}$ 43.5 ± 1.6723	M1		
	$\Rightarrow \quad (41.8, 45.2)$	A1	6	(AWRT)
(ii)	CI contains mean (45)	B1 dep		Must be clear use of 45 and **not** 43.5
	Bishen's belief probably justified	B1 dep		
	or			
	[Since 45 within CI] but close to upper limit, there is some evidence that Bishen's Belief is untrue [but the evidence is not significant at 5 %.]			
	(75% of sample less than 45grams)	(B1)	2	
6(b)(i)	$H_0: \mu = 45$			
	$H_0: \mu < 45$	B1		(both)
	Test statistic: $t = \frac{43.5 - 45}{2/\sqrt{8}}$	M1		
	$= -2.12$	A1		$P(t_7 < -2.12.) = 0.035791$
	$\nu = 7 \ \Rightarrow \ t_{crit} = -1.895$	B1		< 0.05
	$\Rightarrow$ Reject H_0	A1		
	Evidence at the 5% level of significance . to support Abi's claim that **mean** content < 45 grams	E1	6	
(ii)	Type I error	B1		
	have/may have rejected H_0 when H_0 true	B1	2	Clear statement
	or			
	No error	(B1)		
	have/may have accepted H_0 when H_0 true	(B1)		Clear statement
	Total		**16**	
	TOTAL		**75**	