General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MPC1

Unit Pure Core 1

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The line AB has equation $7x + 3y = 13$.

(a) Find the gradient of AB. *(2 marks)*

(b) The point C has coordinates $(-1, 3)$.

(i) Find an equation of the line which passes through the point C and which is parallel to AB. *(2 marks)*

(ii) The point $(1\frac{1}{2}, -1)$ is the mid-point of AC. Find the coordinates of the point A. *(2 marks)*

(c) The line AB intersects the line with equation $3x + 2y = 12$ at the point B. Find the coordinates of B. *(3 marks)*

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2 (a) (i) Express $\sqrt{48}$ in the form $k\sqrt{3}$, where k is an integer. *(1 mark)*

(ii) Simplify $\dfrac{\sqrt{48} + 2\sqrt{27}}{\sqrt{12}}$, giving your answer as an integer. *(3 marks)*

(b) Express $\dfrac{1 - 5\sqrt{5}}{3 + \sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. *(4 marks)*

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3 The volume, $V\,\mathrm{m}^3$, of water in a tank after time t seconds is given by

$$V = \frac{t^3}{4} - 3t + 5$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}t}$. *(2 marks)*

(b) (i) Find the rate of change of volume, in $\mathrm{m}^3\,\mathrm{s}^{-1}$, when $t = 1$. *(2 marks)*

(ii) Hence determine, with a reason, whether the volume is increasing or decreasing when $t = 1$. *(1 mark)*

(c) (i) Find the positive value of t for which V has a stationary value. *(3 marks)*

(ii) Find $\dfrac{\mathrm{d}^2V}{\mathrm{d}t^2}$, and hence determine whether this stationary value is a maximum value or a minimum value. *(3 marks)*

4 (a) Express $x^2 + 5x + 7$ in the form $(x + p)^2 + q$, where p and q are rational numbers. *(3 marks)*

(b) A curve has equation $y = x^2 + 5x + 7$.

(i) Find the coordinates of the vertex of the curve. *(2 marks)*

(ii) State the equation of the line of symmetry of the curve. *(1 mark)*

(iii) Sketch the curve, stating the value of the intercept on the y-axis. *(3 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 5x + 7$. *(3 marks)*

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5 The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 2x^2 + 3$.

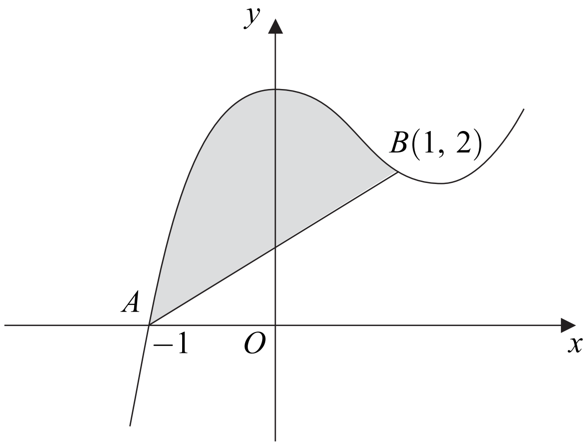
(a) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(b) Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(c) (i) Express $\mathrm{p}(x) = x^3 - 2x^2 + 3$ in the form $(x + 1)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(ii) Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root. *(2 marks)*

6 The curve with equation $y = x^3 - 2x^2 + 3$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and passes through the point $B(1, 2)$.

(a) Find $\int_{-1}^{1} (x^3 - 2x^2 + 3)\, dx$. *(5 marks)*

(b) Hence find the area of the shaded region bounded by the curve $y = x^3 - 2x^2 + 3$ and the line AB. *(3 marks)*

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7 Solve each of the following inequalities:

(a) $2(4 - 3x) > 5 - 4(x + 2)$; *(2 marks)*

(b) $2x^2 + 5x \geqslant 12$. *(4 marks)*

8 A circle has centre $C(3, -8)$ and radius 10.

(a) Express the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$ *(2 marks)*

(b) Find the x-coordinates of the points where the circle crosses the x-axis. *(3 marks)*

(c) The tangent to the circle at the point A has gradient $\frac{5}{2}$. Find an equation of the line CA, giving your answer in the form $rx + sy + t = 0$, where r, s and t are integers. *(3 marks)*

(d) The line with equation $y = 2x + 1$ intersects the circle.

(i) Show that the x-coordinates of the points of intersection satisfy the equation

$$x^2 + 6x - 2 = 0$$ *(3 marks)*

(ii) Hence show that the x-coordinates of the points of intersection are of the form $m \pm \sqrt{n}$, where m and n are integers. *(2 marks)*

END OF QUESTIONS