

# 4724 Core Mathematics 4

1	<p><u>Long Division</u> For leading term <math>3x^2</math> in quotient B1</p> <p>Suff evid of div process (<math>ax^2</math>, mult back, attempt sub) M1</p> <p>(Quotient) = <math>3x^2 - 4x - 5</math> A1</p> <p>(Remainder) = <math>-x + 2</math> A1</p> <p><u>Identity</u> <math>3x^4 - x^3 - 3x^2 - 14x - 8 = Q(x^2 + x + 2) + R</math> *M1</p> <p><math>Q = ax^2 + bx + c, R = dx + e</math> &amp; attempt <math>\geq 3</math> ops. dep*M1 If <math>a = 3</math>, this <math>\Rightarrow</math> 1 operation</p> <p><math>a = 3, b = -4, c = -5</math> A1 dep*M1; <math>Q = ax^2 + bx + c</math></p> <p><math>d = -1, e = 2</math> A1</p> <p><u>Inspection</u> Use 'Identity' method; if <math>R = e</math>, check cf(x) correct before awarding 2<sup>nd</sup> M1</p>	
<b>4</b>		
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2	<p><u>Indefinite Integral</u> Attempt to connect <math>dx</math> &amp; <math>d\theta</math> *M1 Incl <math>\frac{dx}{d\theta}</math> or <math>\frac{d\theta}{dx}</math>; not <math>dx = d\theta</math></p> <p>Reduce to <math>\int 1 - \tan^2 \theta (d\theta)</math> A1 A0 if <math>\frac{d\theta}{dx} = \sec^2 \theta</math>; but allow all following</p> <p>Use <math>\tan^2 \theta = (1, -1) + (\sec^2 \theta, -\sec^2 \theta)</math> dep*M1 A marks</p> <p>Produce <math>\int 2 - \sec^2 \theta (d\theta)</math> A1</p> <p>Correct <math>\sqrt{\quad}</math> integration of function of type <math>d + e \sec^2 \theta</math> <math>\sqrt{A1}</math> including <math>d = 0</math></p> <p>EITHER Attempt limits change (allow degrees here) M1 (This is 'limits' aspect; the</p> <p>OR Attempt integ, re-subst &amp; use original (<math>\sqrt{3}, 1</math>) integ need not be accurate)</p> <p><math>\frac{1}{6}\pi - \sqrt{3} + 1</math> isw Exact answer required A1</p>	
<b>7</b>		

3 (i)  $(1 + \frac{x}{a})^{-2} = 1 + (-2)\frac{x}{a} + \frac{-2 \cdot -3}{2}(\frac{x}{a})^2 + \dots$  M1 Check 3<sup>rd</sup> term; accept  $\frac{x^2}{a}$   
 $= 1 - \frac{2x}{a} + \dots$  or  $1 + (-\frac{2x}{a})$  B1 or  $1 - 2xa^{-1}$  (Ind of M1)  
 $\dots + \frac{3x^2}{a^2} + \dots$  (or  $3(\frac{x}{a})^2$  or  $3x^2 a^{-2}$ ) A1 Accept  $\frac{6}{2}$  for 3  
 $(a+x)^{-2} = \frac{1}{a^2} \left\{ \text{their expansion of } (1 + \frac{x}{a})^{-2} \right\}$  mult out  $\sqrt{A1}$  4  $\frac{1}{a^2} - \frac{2x}{a^3} + \frac{3x^2}{a^4}$ ; accept eg  $a^{-2}$

(ii) Mult out  $(1-x)$ (their exp) to produce all terms/cfs ( $x^2$ ) M1 Ignore other terms  
 Produce  $\frac{3}{a^2} + \frac{2}{a} (= 0)$  or  $\frac{3}{a^4} + \frac{2}{a^3} (= 0)$  or AEF A1 Accept  $x^2$  if in both terms  
 $a = -\frac{3}{2}$  www seen anywhere in (i) or (ii) A1 3 Disregard any ref to  $a = 0$

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4 (i) Differentiate as a product,  $u dv + v du$  M1 or as 2 separate products  
 $\frac{d}{dx}(\sin 2x) = 2 \cos 2x$  or  $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$  B1  
 $e^x(2 \cos 2x + 4 \sin 2x) + e^x(\sin 2x - 2 \cos 2x)$  A1 terms may be in diff order  
 Simplify to  $5 e^x \sin 2x$  www A1 4 Accept  $10e^x \sin x \cos x$

(ii) Provided result (i) is of form  $k e^x \sin 2x$ ,  $k$  const  
 $\int e^x \sin 2x dx = \frac{1}{k} e^x (\sin 2x - 2 \cos 2x)$  B1  
 $[e^x (\sin 2x - 2 \cos 2x)]_0^{\frac{1}{4}\pi} = e^{\frac{1}{4}\pi} + 2$  B1  
 $\frac{1}{5} \left( e^{\frac{1}{4}\pi} + 2 \right)$  B1 3 Exact form to be seen

**SR** Although 'Hence', award M2 for double integration by parts and solving + A1 for correct answer.

7

5 (i)  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$  aef used M1

$= \frac{4t + 3t^2}{2 + 2t}$  A1

Attempt to find  $t$  from one/both equations M1 or diff (ii) cartesian eqn  $\rightarrow$  M1

State/imply  $t = -3$  is only solution of both equations A1 subst (3,-9), solve for  $\frac{dy}{dx} \rightarrow$  M1

Gradient of curve =  $-\frac{15}{4}$  or  $\frac{-15}{4}$  or  $\frac{15}{-4}$  A1 **5** grad of curve =  $-\frac{15}{4} \rightarrow$  A1

[**SR** If  $t = 1$  is given as solution & not disqualified, award A0 +  $\sqrt{A1}$  for grad =  $-\frac{15}{4}$  &  $\frac{7}{4}$ ;  
If  $t = 1$  is given/used as only solution, award A0 +  $\sqrt{A1}$  for grad =  $\frac{7}{4}$ ]

(ii)  $\frac{y}{x} = t$  B1

Substitute into either parametric eqn M1

Final answer  $x^3 = 2xy + y^2$  A2 **4**

[**SR** Any correct unsimplified form (involving fractions or common factors)  $\rightarrow$  A1]

9

6 (i)  $4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$  M1

$A = 5$  A1 'cover-up' rule, award B1

$B = -5$  A1

$C = -6$  A1 **4** 'cover-up' rule, award B1

Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1

(ii)  $\int \frac{A}{x-5} dx = A \ln(5-x)$  or  $A \ln|5-x|$  or  $A \ln|x-5|$   $\sqrt{B1}$  but not  $A \ln(x-5)$

$\int \frac{B}{x-3} dx = B \ln(3-x)$  or  $B \ln|3-x|$  or  $B \ln|x-3|$   $\sqrt{B1}$  but not  $B \ln(x-3)$

If candidate is awarded B0,B0, then award **SR**  $\sqrt{B1}$  for  $A \ln(x-5)$  **and**  $B \ln(x-3)$

$\int \frac{C}{(x-3)^2} dx = -\frac{C}{x-3}$   $\sqrt{B1}$

$5 \ln \frac{3}{4} + 5 \ln 2$  aef, isw  $\sqrt{A \ln \frac{3}{4} - B \ln 2}$   $\sqrt{B1}$  Allow if **SR** B1 awarded

$-3$   $\sqrt{\frac{1}{2}C}$   $\sqrt{B1}$  **5**

[Mark at earliest correct stage & isw; no ln 1]

9

- 7 (i) Attempt scalar prod  $\{\mathbf{u} \cdot (4\mathbf{i} + \mathbf{k})$  or  $\mathbf{u} \cdot (4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})\} = 0$  M1 where  $\mathbf{u}$  is the given vector
- Obtain  $\frac{12}{13} + c = 0$  or  $\frac{12}{13} + 3b + 2c = 0$  A1
- $c = -\frac{12}{13}$  A1
- $b = \frac{4}{13}$  A1 cao No ft
- Evaluate  $\left(\frac{3}{13}\right)^2 + (\text{their } b)^2 + (\text{their } c)^2$  M1 Ignore non-mention of  $\sqrt{\quad}$
- Obtain  $\frac{9}{169} + \frac{144}{169} + \frac{16}{169} = 1$  AG A1 6 Ignore non-mention of  $\sqrt{\quad}$

- (ii) Use  $\cos \theta = \frac{\mathbf{x} \cdot \mathbf{y}}{|\mathbf{x}||\mathbf{y}|}$  M1
- Correct method for finding scalar product M1
- $36^\circ$  (35.837653...) Accept 0.625 (rad) A1 3 From  $\frac{18}{\sqrt{17}\sqrt{29}}$
- SR If  $4\mathbf{i} + \mathbf{k} = (4, 1, 0)$  in (i) & (ii), mark as scheme but allow final A1 for  $31^\circ$  (31.160968) or 0.544

**9**

- 8 (i)  $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$  B1
- $\frac{d}{dx}(uv) = u \, dv + v \, du$  used on  $(-7)xy$  M1
- $\frac{d}{dx}(14x^2 - 7xy + y^2) = 28x - 7x \frac{dy}{dx} - 7y + 2y \frac{dy}{dx}$  A1 (= 0)
- $2y \frac{dy}{dx} - 7x \frac{dy}{dx} = 7y - 28x \rightarrow \frac{dy}{dx} = \frac{28x - 7y}{7x - 2y}$  www AG A1 4 As AG, intermed step nec

- (ii) Subst  $x = 1$  into eqn curve & solve quadratic eqn in  $y$  M1 ( $y = 3$  or  $4$ )
- Subst  $x = 1$  and (one of) their  $y$ -value(s) into given  $\frac{dy}{dx}$  M1  $\left(\frac{dy}{dx} = 7 \text{ or } 0\right)$
- Find eqn of tgt, with their  $\frac{dy}{dx}$ , going through (1, their  $y$ ) \*M1 using (one of)  $y$  value(s)
- Produce either  $y = 7x - 4$  or  $y = 4$  A1
- Solve simultaneously their two equations dep\*M1 provided they have two
- Produce  $x = \frac{8}{7}$  A1 6

**10**

9 (i)  $\frac{20}{k_1}$  (seconds) B1 1

(ii)  $\frac{d\theta}{dt} = -k_2(\theta - 20)$  B1 1

- (iii) Separate variables or invert each side M1 Correct eqn or very similar  
 Correct int of each side (+ c) A1,A1 for each integration  
 Subst  $\theta = 60$  when  $t = 0$  into eqn containing 'c' M1 or  $\theta = 60$  when  $t =$  their (i)  
 $c$  (or  $-c$ ) =  $\ln 40$  or  $\frac{1}{k_2} \ln 40$  or  $\frac{1}{k_2} \ln 40k_2$  A1 Check carefully their 'c'  
 Subst their value of  $c$  and  $\theta = 40$  back into equation M1 Use scheme on LHS  
 $t = \frac{1}{k_2} \ln 2$  A1 Ignore scheme on LHS  
 Total time =  $\frac{1}{k_2} \ln 2 +$  their (i) (seconds)  $\sqrt{A1}$  8

**SR** If the negative sign is omitted in part (ii), allow all marks in (iii) with  $\ln 2$  replaced by  $\ln \frac{1}{2}$ .

**SR** If definite integrals used, allow M1 for eqn where  $t = 0$  and  $\theta = 60$  correspond; a second M1 for eqn where  $t = t$  and  $\theta = 40$  correspond & M1 for correct use of limits. Final answer scores 2.