Mark Scheme (Final)

Summer 2007

GCE

GCE Mathematics (6686/01)

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6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
1. a	d: 14 2 18 25 0 -8 4 4 12 20	M1
	$\bar{d} = \pm 9.1$ sd = $\sqrt{106.7} = 10.332..$ $(\sum d = 91, \quad \sum x^2 = 1789)$	A1 A1
	$H_0 : \mu_d = 0 \quad H_1 : \mu_d \neq 0$	B1
	$t = \pm\frac{9.1\sqrt{10}}{10.332} = \pm\ 2.785$ awrt ± 2.78 or 2.79	M1 A1
	Critical value $t_9 = \pm 1.833$	B1
	Significant. There is a difference between blood pressure measured by arm cuff and finger monitor.	A1
		(8)
b.	The difference in measurements of blood pressure is normally distributed	B1
		(1)
	Notes. (a) One tail test Loses the first B1 . CV is 1.383 in this case. Can get 7/8 (b) looking for the difference in measurements. Not just it is normally distributed.	

Question Number	Scheme		Marks
2. a)	$\mathrm{E}(\overline{X}) = \mu$		B1
	$\mathrm{Var}(\overline{X}) = \mathrm{Var}\left(\dfrac{X_1 + X_2 + X_3 + \ldots. + X_n}{n}\right)$		
	$= \dfrac{\sigma^2}{n}$		B1
			(2)
b)	$\mathrm{E}(U) = \dfrac{1}{n+m}\left(n\mathrm{E}(\overline{\mathrm{X}}) + m\mathrm{E}(\overline{\mathrm{Y}})\right)$		M1
	$= \dfrac{1}{n+m}(n\mu + m\mu)$		A1
	$= \mu \quad \underline{\Rightarrow U \text{ is unbiased}}$	state unbiased	A1
			(3)
c)	$\mathrm{Var}(\overline{Y}) = \dfrac{\sigma^2}{m}$		B1
	$\mathrm{Var}(U) = \dfrac{n^2\mathrm{Var}(\overline{X}) + m^2\mathrm{Var}(\overline{Y})}{(n+m)^2}$		M1
	$= \dfrac{n^2\dfrac{\sigma^2}{n} + m^2\dfrac{\sigma^2}{m}}{(n+m)^2}$		A1
	$= \dfrac{n\sigma^2 + m\sigma^2}{(n+m)^2}$		
	$= \dfrac{\sigma^2}{n+m} \quad *$	cso	A1
			(4)
d)	$\dfrac{n\overline{X} + m\overline{Y}}{n+m}$ is a better estimate since variance is smaller.		B1 B1
			(2)

Question Number	Scheme	Marks
3. a	$H_0 : \sigma^2_F = \sigma^2_M \qquad H_1 : \sigma^2_F \neq \sigma^2_M$	B1
	$s^2_F = \frac{1}{6}(17956.5 - 7 \times 50.6^2) = \frac{33.98}{6} = 5.66333....$	B1
	$s^2_M = \frac{1}{9}(28335.1 - 10 \times 53.2^2) = \frac{32.7}{9} = 3.63333....$	B1
	$\frac{s^2{}_F}{s^2{}_M} = 1.5587...$ (Reciprocal 0.6415)	M1 A1
	$F_{6,9} = 3.37$ (or 0.24)	B1
	Not in critical region. <u>Variances</u> of the two distributions <u>are the same</u>	A1
		(7)
b.	$H_0 : \mu_F = \mu_M \qquad H_1 : \mu_F < \mu_M$	B1
	Pooled estimate $s^2 = \frac{6 \times 5.66333.... + 9 \times 3.63333}{15}$	M1
	$= 4.44533$	
	$s = 2.11$	
	$t = \frac{50.6 - 53.2}{2.11\sqrt{\frac{1}{7} + \frac{1}{10}}} = \pm 2.50$	M1 A1
	C.V. $t_{15}(5\%) = \pm 1.753$	B1
	Significant. The mean length of the <u>females forewing</u> is less than the length of the males forewing	A1
		(6)
	Notes (a) need to have <u>variance</u> and <u>the same</u> o.e (b) need female and forewing(wing)	

Question Number	Scheme	Marks
4.a)	$H_0 : \sigma^2 = 0.9$ $H_1 : \sigma^2 \neq 0.9$	B1
	$\nu = 19$	
	CR (Lower tail 10.117)	B1
	Upper tail 30.144	B1
	Test statistic = $\frac{19 \times 1.5}{0.9} = 31.6666$, significant	M1 A1
	There is sufficient evidence that the <u>variance</u> of the length of spring is <u>different to 0.9</u>	A1
		(6)
b)	$H_0 : \mu = 100$ $H_1 : \mu > 100$	B1
	$t_{19} = 1.328$	B1
	$t = \frac{100.6 - 100}{\sqrt{\frac{1.5}{20}}} = 2.19$	M1 A1 A1
	Significant. The mean <u>length of spring</u> is <u>greater than 100</u>	B1
		(6)
	Notes (a) only need to see 30.144 need variance in conclusion (b) conclusion must be in context. Length of spring needed	

Question Number	Scheme	Marks
5.a)	Power = P $(X \le 3 / \lambda)$	M1
	$= e^{-\lambda} + e^{-\lambda}\lambda + \frac{e^{-\lambda}\lambda^2}{2} + \frac{e^{-\lambda}\lambda^3}{6}$	A1
	$= \frac{e^{-\lambda}}{6}(6 + 6\lambda + 3\lambda^2 + \lambda^3)$	A1 **(3)**
b)	CR is $X \le 3$	M1
	Size = P$[X \le 3 / \lambda = 7]$	
	= 0.0818	A1 **(2)**
c)	P(Type II error) = 1 – power	
	$= 1 - \frac{e^{-4}}{6}(6 + 6\times 4 + 3\times 4^2 + 4^3)$	M1
	= 0.5665..	A1 **(2)**
6.a)	$\frac{\overline{X} - 250}{\frac{4}{\sqrt{15}}} > 2.3263$ or $\frac{\overline{X} - 250}{\frac{4}{\sqrt{15}}} < -2.3263$ ± 2.3262	B1 M1
	$\overline{X} > 252.40...$ or $\overline{X} < 247.6...$ awrt 252 and 248	A1 **(3)**
b)	$P(\overline{X} < 252.4 / \mu = 254)$ - $P(\overline{X} < 247.6 / \mu = 254)$ using their '252.4' and '247.6	M1
	$= P\left(Z < \frac{252.4 - 254}{\frac{4}{\sqrt{15}}}\right) - P\left(Z < \frac{247.6 - 254}{\frac{4}{\sqrt{15}}}\right)$ stand using 4/√15, 254 their '252.4' or '247.6	M1
	= P(Z < -1.5492) – P(Z < -6.20) -1.5492 and -6.20 o.e.	A1
	= (1 – 0.9394) – (1 – 1)	M1
	= 0.0606	A1 **(5)**
	Notes (a) only needs to try and find one side for M1 (b) only need to see one of the standardisation for second M1 if consider only 252.4 and get 0.0606 they get M0 M1 A0 M1 A1 ie they can get 3/5	

Question Number	Scheme		Marks
7.	$\bar{x} = 4.01$		B1
	$s = 0.7992\ldots$		M1 A1
(a)	$4.01 \pm t_9\,(2.5\%)\dfrac{0.7992..}{\sqrt{10}} = 4.01 \pm 2.262\dfrac{0.7992..}{\sqrt{10}}$	2.262	B1
		their $\bar{x}$ and s and $\sqrt{10}$	M1 A1√
	$= 4.5816\ldots$ and $3.4383\ldots$	awrt 4.58 and 3.44	A1 **(7)**
(b)	$2.700 < \dfrac{9\times 0.7992..^2}{s^2} < 19.023$	2.7,19.023	B1 B1
		$9\times s^2/\sigma^2$	M1
	$\sigma^2 < 2.13, \quad \sigma^2 > 0.302$	both awrt 2.13, 0.302	A1 **(4)**
(c)	$\mathrm{P}(X > 7) = \mathrm{P}\left(Z > \dfrac{7-\mu}{\sigma}\right)$ needs to be as high as possible		M1
	Therefore μ and σ must be as big as possible		M1
	$= \mathrm{P}\left(Z > \dfrac{7-4.581}{\sqrt{2.13}}\right)$		A1√
	$= 1 - 0.9515$		
	$= 0.0485$		
	$= 4.85\%$	4.8 to 4.9	A1 **(4)**
	Notes (a) $s^2 = 0.63877\ldots$ (c) M1 may be implied by them using their highest μ and σ.		