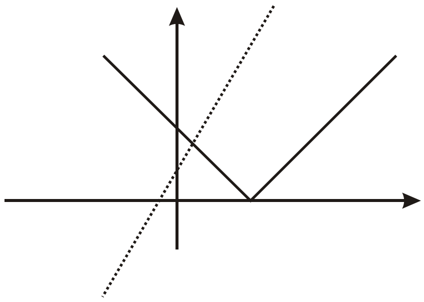
1. (a)



Shape, vertex on x-axis B1

At least 2a seen on positive x-axis B1 2

(b) Attempting to solve $-(x - 2a) = 2x + a$ anywhere M1
Completely correct method dep M1
[e.g. solving $-(x - 2a) > 2x + a$;
if finding two "solutions" needs to be evidence for giving "correct" result]
$x < 1/3\, a$ A1 3

[5]

2. I.F. = $e^{\int 2\cot 2x\,dx}$; $= \sin 2x$ M1 A1

Multiplying **throughout** by IF. M1(*)

$Y \times$ (IF) = integral of candidate's RHS M1

$= \int 2\sin^2 x\cos x\,dx$ or $\int -\left(\frac{\cos 3x - \cos x}{2}\right)dx$ M1

[This M gained when in position to complete integration, dep on M(*)]

$= \frac{2}{3}\sin^3 x(+C)$ or $-\frac{1}{6}\sin 3x + \frac{1}{2}\sin x + c$ A1

$y = \frac{2\sin^3 x}{3\sin 2x} + \frac{C}{\sin 2x}$ or $-\frac{\sin 3x}{6\sin 2x} + \frac{\sin x}{2\sin 2x} + \frac{c}{\sin 2x}$ or equiv. A1ft

[7]

3. (a) $\frac{dy}{dx} = x\frac{dv}{dx} + v$, $\frac{d^2y}{dx^2} = x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}$ M1A1

[M1 for diff. product, A1 both correct]

$\therefore x^2\left(x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}\right) - 2x\left(x\frac{dv}{dx} + v\right) + (2 + 9x^2)vx = x^5$ M1

$x^3\frac{d^2v}{dx^2} + 2x^2\frac{dv}{dx} - 2x^2\frac{dv}{dx} - 2vx + 2vx + 9vx^3 = x^5$ A1

$[x^3\frac{d^2v}{dx^2} + +9vx^3 = x^5]$

Given result: $\frac{d^2v}{dx^2} + 9v = x^2$ A1 5

cso

(b) CF: $v = A\sin 3x + b\cos 3x$ (may just write it down) M1A1

Appropriate form for P1: $v = \lambda x^2 + \mu$ (or $ax^2 + bx + c$) M1

Complete method to find λ and μ (or a, b, c) M1

$v = A\sin 3x + B\cos 3x + \frac{1}{9}x^2 - \frac{2}{81}$ M1A1ft6

[f.t. only on wrong CF]

(c) $\therefore y = Ax\sin 3x + Bx\cos 3x + \frac{1}{9}x^3 - \frac{2}{81}x$ B1ft 1

[f.t. for y = x (candidate's CF + PI), providing two arbitrary constants]

[12]

4. (a) For C: Using polar/ Cartesian relationships to form Cartesian equation M1

so $x^2 + y^2 = 6x$ A1

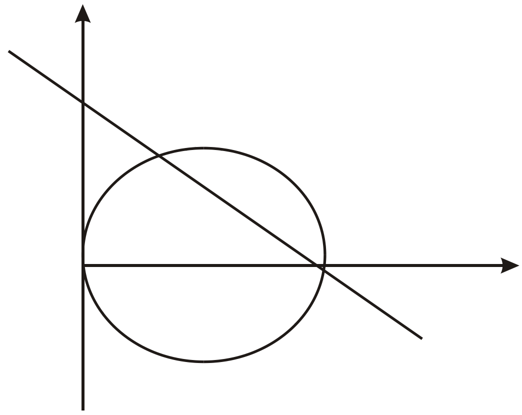
[Equation in any form: e.g. $(x - 3)^2 + y^2 = 9$ from sketch.

or $\sqrt{x^2 + y^2} = \frac{6x}{\sqrt{x^2 + y^2}}$]

For D: $r\cos\left(\frac{\pi}{3} - \theta\right) = 3$ and attempt to expand M1

$\frac{x}{2} + \frac{\sqrt{3}y}{2} = 3$ (any form) M1A1 5

(b)



"Circle", symmetric in initial line passing through pole	B1	
Straight line	B1	
Both passing through (6, 0)	B1	3

(c)	Polars: Meet where $6\cos\theta\cos(\frac{\pi}{3} - \theta) = 3$	M1	
	$\sqrt{3}\sin\theta\cos\theta = \sin^2\theta$	M1	
	$\sin\theta = 0$ or $\tan\theta = \sqrt{3}$ $[\theta = 0$ or $\frac{\pi}{3}]$	M1	
	Points are (6, 0) and $(3, \frac{\pi}{3})$	B1, A1	5
		[13]	

Alternatives (only more common):

(a)	Equation of D:	
	Finding two points on line	M1
	Using correctly in Cartesian equation for straight line	M1
	Correct Cartesian equation	A1

(c)	Cartesian: Eliminate x or y to form quadratic in one variable	M1
	$[2x^2 - 15x + 18 = 0,\ 4y^2 - 6\sqrt{3}\ y = 0]$	
	Solve to find values of x or y	M1
	Substitute to find values of other variable	
	$\left[x = \frac{3}{2} \text{ or } 6; \quad y = 0 \text{ or } \frac{3\sqrt{3}}{2}\right]$	B1A1
	Points must be (6, 0) and $(3, \frac{\pi}{3})$	B1A1

5. $\dfrac{dy}{dx}+\dfrac{2}{1+x}y=\dfrac{1}{x(x+1)}$ M1

Attempt $y' = Py = Q$ form

I.F. $= e^{\int \frac{2}{1+x}dx} = e^{2\ln(1+x)}$, $= (1+x)^2$ M1, A1

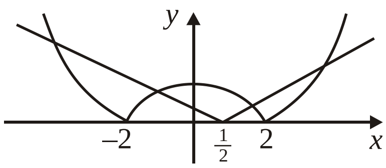
$\therefore y(1+x)^2 = \int\left(\dfrac{x+1}{x}\right)dx$ <u>OR</u> $\dfrac{d}{dx}(y(1+x)^2) = \dfrac{x+1}{x}$ M1 (ft I.F.)

i.e. $(y(1+x)^2 =)x + \ln x + (C)$ M1 A1

$\underline{y=}\,\dfrac{x+\ln x+C}{(1+x)^2}$ A1 c.a.o.

[7]

6. (a)



 shape – Symmetric about y-axis B1

 shape – Vertex on positive x-axis B1

–z, z B1

$\dfrac{1}{2}$ B1 4

(b) $x^2 - 4 = 2x - 1$ M1

$x^2 - 2x - 3 = 0 \Rightarrow \underline{x = 3, -1}$ A1

$x^2 - 4 = -(2x - 1)$ M1

$x^2 + 2x - 5 = 0, \Rightarrow x = \dfrac{-2 \pm \sqrt{4+20}}{2}$ A1,

$x = \underline{-1 \pm}\sqrt{6}$ A1 5

correct 3 term quadratic = 0

(c) $x < -1 - \sqrt{6}$; $-1 < x < \sqrt{6} - 1$, $x > 3$ (√surds) B1ft; B1ft; B1

Accept 3sf.

[12]

7. (a) $2m^2 + 5m + 2 = 0$ M1

Attempt aux eqn $\rightarrow m =$

$\Rightarrow m = -\frac{1}{2}, -2$

$\therefore x_{CF} = Ae^{-2t} + Be^{-\frac{1}{2}t}$ A1

C.F.

Particular Integral: $x = pt + q$ B1

P.I.

$\dot{x} = p$, $\ddot{x} = 0$ and sub. M1

$\Rightarrow 5p + 2q + 2pt = 2t + q \rightarrow \underline{p = 1, q = 2}$ A1

General solution $x = \underline{Ae^{-2t} + Be^{-\frac{1}{2}t} + t + 2}$ A1ft (ft ms, p.q)

(b) $x = 3, t = 0 \Rightarrow 3 = A + B + 2$ (or $A + B = 1$) M1

$\dot{x} = -2Ae^{-2t} - \frac{1}{2}Be^{-\frac{1}{2}t} + 1$ M1

Attempt $\dot{x}$

$\dot{x} = -1, t = 0 \Rightarrow -1 = -2A - \frac{1}{2}B + 1$ (or $4A + B = 4$) A1

2 correct eqns

Solving $\rightarrow A = 1, B = 0$ and $\underline{x = e^{-2t} + t + 2}$ A1 4

(c) $\dot{x} = -2e^{-2t} + 1 = 0$ M1

$\dot{x} = 0$

$\Rightarrow t = \frac{1}{2}\ln 2$ A1

$\ddot{x} = 4e^{-2t} > 0\ (\forall t)\ \therefore$ min M1

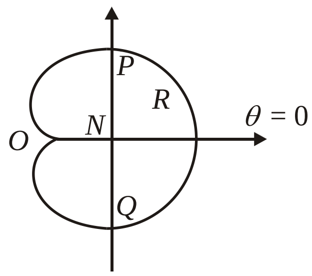
Min $x = e^{-\ln 2} + \frac{1}{2}\ln 2 + 2$

$= \frac{1}{2} + \frac{1}{2}\ln 2 + 2$

$\underline{= \frac{1}{2}(5 + \ln 2)}$ (*) A1 c.s.o.

[14]

8. (a)



$4a(1+\cos\theta) = \dfrac{3a}{\cos\theta}$ *or* $r = 4a\left(1+\dfrac{3a}{r}\right)$ M1

$4\cos^2\theta + 4\cos\theta - 3 = 0$ *or* $r^2 - 4ar - 12a^2 = 0$ A1

$(2\cos\theta - 1)(2\cos\theta + 3) = 0$ *or* $(r-6a)(r+2a) = 0$ M1

$\cos\theta = \dfrac{1}{2}$, $\left(\theta = \dfrac{\pi}{3}\right)$ *or* $r = 6a$ A1

Note ON = 3a

$PQ = 2 \times ON \tan\dfrac{\pi}{6} = 6\sqrt{3}a$ (*) cso M1 A1

or $PQ = 2\times\sqrt{[(6a)^2 - (3a)^2]} = 2\sqrt{(27a^2)} = 6\sqrt{3}a$ (*) cso

or any complete equivalent

(b) $2 \times \dfrac{1}{2}\int_0^{\pi/3} r^2 \, d\theta = \ldots\int_{\ldots}^{\ldots} 16a^2(1+\cos\theta)^2 d\theta$ M1

$\int r^2 d\theta$

$= \ldots\int_{\ldots}^{\ldots}\left(1 + 2\cos\theta + \dfrac{1}{2} + \dfrac{1}{2}\cos 2\theta\right)d\theta$ M1

$\cos^2\theta \rightarrow \cos 2\theta$

$= \ldots\left[\dfrac{3}{2}\theta + 2\sin\theta + \dfrac{1}{4}\sin 2\theta\right]$ A1

$= 16a^2\left[\dfrac{\pi}{2} + \sqrt{3} + \dfrac{\sqrt{3}}{8}\right]$ $(= 2a^2[4\pi + 9\sqrt{3}] \approx 56.3a^2)$ M1 A1

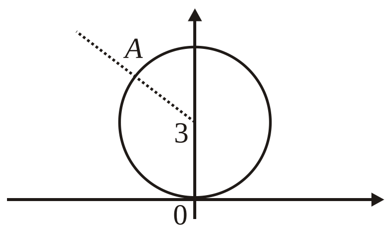
use of their $\dfrac{\pi}{3}$ for M1

Area of $\Delta POQ = \dfrac{1}{2}6\sqrt{3}\,a \times 3a$ or $9a^2\sqrt{3}$ B1

$R = a^2(8\pi + 9\sqrt{3})$ cao A1 7

[13]

9. (a)



Circle | M1
Correct circle. | A1 | 2
(centre (0, 3), radius 3)

(b) Drawing correct **half-**line passing as shown | B1

Find either x or y coord of A. | M1A1

$z = -\dfrac{3\sqrt{2}}{2} + \left(3 + \dfrac{3\sqrt{2}}{2}\right)\mathrm{i}$ | A1 | 4

[Algebraic approach, i.e. using $y = 3 - x$ and equation of circle will only gain M1A1, unless the second solution is ruled out, when B1 can be given by implication, and final A1, if correct]

(c) $|z - 3\mathrm{i}| = 3 \rightarrow \left|\dfrac{2\mathrm{i}}{\omega} - 3\mathrm{i}\right| = 3$ | M1

$\Rightarrow \dfrac{|2\mathrm{i} - 3\mathrm{i}\omega|}{|\omega|} = 3$ | A1

$\Rightarrow |\, \omega - {}^{2}/_{3} \,| = |\, \omega \,|$ | M1A1

Line with equation $u = {}^{1}/_{3}$ ($x = {}^{1}/_{3}$) | A1 | 5

[11]

Some alternatives:

(i) $\omega = \dfrac{2\mathrm{i}}{x + \mathrm{i}y} = \dfrac{2\mathrm{i}(x - \mathrm{i}y)}{x^2 + y^2} \Rightarrow u = \dfrac{2y}{x^2 + y^2},\ v = \dfrac{2x}{x^2 + y^2}$ | M1A1

As $x^2 + y^2 - 6y = 0$, $u = \dfrac{1}{3}$ | M1, A1A1

(ii) $\omega = \dfrac{2\mathrm{i}}{3\cos\theta + 3\mathrm{i}(1 + \sin\theta)} = \dfrac{2\mathrm{i}\{\cos\theta - \mathrm{i}(1 + \sin\theta)\}}{3\{\cos^2\theta + (1 + \sin\theta)^2\}}$ | M1A1

$= \dfrac{2}{3}\,\dfrac{(1 + \sin\theta) + \mathrm{i}\cos\theta}{2 + 2\sin\theta},\ = \dfrac{1}{3} + \mathrm{i}\,\dfrac{\cos\theta}{1 + \sin\theta},$ | M1A1

So locus is line $u = \dfrac{1}{3}$ | A1

10.	(a)	$z^n = e^{in\theta} = (\cos n\theta + i \sin n\theta)$, $z^{-n} = e^{-in\theta} = \frac{1}{z^n}(\cos n\theta - i \sin n\theta)$	M1	
		Completion (needs to be convincing) $z^n - \frac{1}{z^n} = 2i \sin n\theta$ (*) AG	A1	2
	(b)	$\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$	M1A1	
		$= \left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$		
		$(2 i\sin\theta)^5 = 32i \sin^5 \theta = 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin \theta$	M1A1	
		$\Rightarrow \sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10 \sin \theta)$ (*) AG	A1	5
	(c)	Finding $\sin^5 \theta = ¼ \sin\theta$	M1	
		$\theta = 0, \pi$ (both)	B1	
		$(\sin^4 \theta = ¼) \Rightarrow \sin \theta = \pm \frac{1}{\sqrt{2}}$	M1	
		$\theta = \frac{\pi}{4}, \frac{3\pi}{4}; \frac{5\pi}{4}, \frac{7\pi}{4}$	A1;A1	5
			[12]	
11.	(a)	$\left(\frac{d^2y}{dx^2}\right)_0 = \frac{1}{4}$	B1	1
	(b)	$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h} \Rightarrow \frac{1}{2} \approx \frac{y_1 - y_{-1}}{0.2} \Rightarrow y_1 - y_{-1} \approx 0.1$	M1A1	
		$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2} \Rightarrow \frac{1}{4} \approx \frac{y_1 - 2 + y_{-1}}{0.01}$	M1	
		$\Rightarrow y_1 + y_{-1} \approx 2.0025$	A1	
		Adding to give $y_1 \approx 1.05125$	M1A1	6
	(c)	Diff: $4(1 + x^2)\frac{d^3y}{dx^3} + 8x\frac{d^2y}{dx^2} + 4x\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = \frac{dy}{dx}$	M1A1	
		Substituting appropriate vales $\Rightarrow 4\left(\frac{d^3y}{dx^3}\right)_0 = -\frac{3}{2} \Rightarrow \left(\frac{d^3y}{dx^3}\right)_0 = -\frac{3}{8}$	M1A1	4
	(d)	$y = y_0 + y'_0 x + \frac{y''_0}{2!}x^2 + \frac{y'''_0}{3!}x^3 + \ldots = 1 + \frac{1}{2}x + \frac{1}{8}x^2 - \frac{1}{16}x^3 + \ldots$	M1A1ft	2
	(e)	1.05119	A1	1
			[14]	