



**ADVANCED SUBSIDIARY GCE
MATHEMATICS**

4725/01

Further Pure Mathematics 1

MONDAY 2 JUNE 2008

Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of 4 printed pages.

2

1 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ and \mathbf{I} is the 2×2 identity matrix. Find

(i) $\mathbf{A} - 3\mathbf{I}$, [2]

(ii) \mathbf{A}^{-1} . [2]

2 The complex number $3 + 4i$ is denoted by a .

(i) Find $|a|$ and $\arg a$. [2]

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = |a|$, [2]

(b) $\arg(z - 3) = \arg a$. [3]

3 (i) Show that $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$. [2]

(ii) Hence find an expression, in terms of n , for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}. \quad [4]$$

4 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \geq 1$,

$$\mathbf{A}^n = \begin{pmatrix} 3^n & \frac{1}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}. \quad [6]$$

5 Find $\sum_{r=1}^n r^2(r-1)$, expressing your answer in a fully factorised form. [6]

6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a , b and c are real, has roots $(3 + i)$ and 2 .

(i) Write down the other root of the equation. [1]

(ii) Find the values of a , b and c . [6]

3

7 Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$, [1]

(ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, [2]

(iii) $\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, [2]

(iv) $\begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$. [2]

8 The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β . Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]

9 (i) Use an algebraic method to find the square roots of the complex number $5 + 12i$. [5]

(ii) Find $(3 - 2i)^2$. [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0$. [4]

10 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$. The matrix \mathbf{B} is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

(i) Show that \mathbf{AB} is non-singular. [2]

(ii) Find $(\mathbf{AB})^{-1}$. [4]

(iii) Find \mathbf{B}^{-1} . [5]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (OCR) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

OCR is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.