



**ADVANCED GCE UNIT**  
**MATHEMATICS**

Further Pure Mathematics 2  
**THURSDAY 7 JUNE 2007**

**4726/01**

Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)  
List of Formulae (MF1)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

**ADVICE TO CANDIDATES**

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

## 2

- 1 The equation of a curve, in polar coordinates, is

$$r = 2 \sin 3\theta, \quad \text{for } 0 \leq \theta \leq \frac{1}{3}\pi.$$

Find the exact area of the region enclosed by the curve between  $\theta = 0$  and  $\theta = \frac{1}{3}\pi$ . [4]

- 2 (i) Given that  $f(x) = \sin(2x + \frac{1}{4}\pi)$ , show that  $f(x) = \frac{1}{2}\sqrt{2}(\sin 2x + \cos 2x)$ . [2]

(ii) Hence find the first four terms of the Maclaurin series for  $f(x)$ . [You may use appropriate results given in the List of Formulae.] [3]

- 3 It is given that  $f(x) = \frac{x^2 + 9x}{(x-1)(x^2+9)}$ .

(i) Express  $f(x)$  in partial fractions. [4]

(ii) Hence find  $\int f(x) dx$ . [2]

- 4 (i) Given that

$$y = x\sqrt{1-x^2} - \cos^{-1}x,$$

find  $\frac{dy}{dx}$  in a simplified form. [4]

(ii) Hence, or otherwise, find the exact value of  $\int_0^1 2\sqrt{1-x^2} dx$ . [3]

- 5 It is given that, for non-negative integers  $n$ ,

$$I_n = \int_1^e (\ln x)^n dx.$$

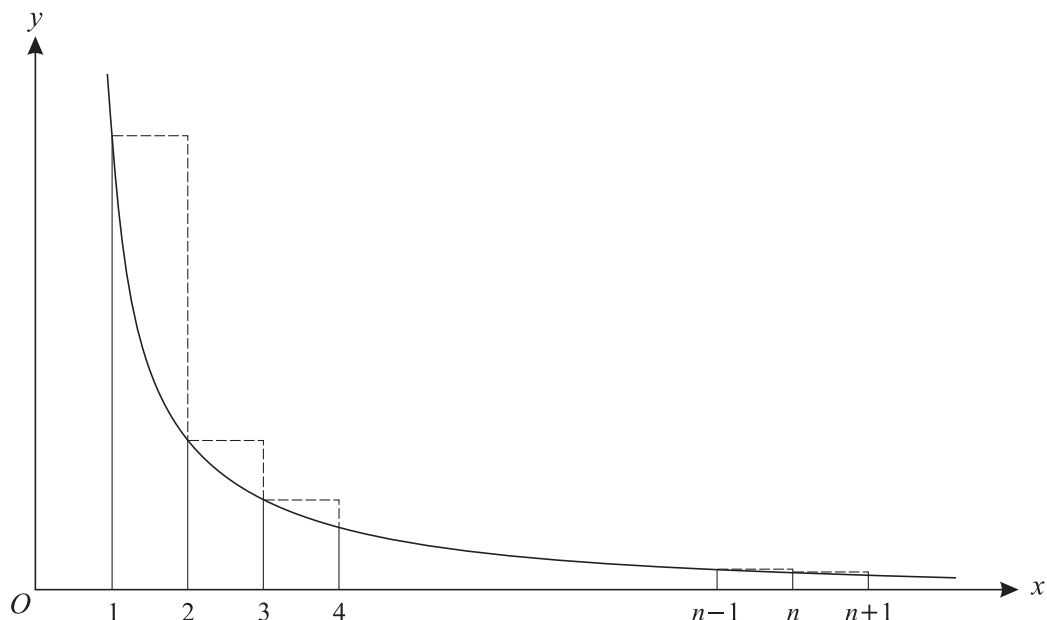
(i) Show that, for  $n \geq 1$ ,

$$I_n = e - nI_{n-1}. \quad [4]$$

(ii) Find  $I_3$  in terms of  $e$ . [4]

3

6



The diagram shows the curve with equation  $y = \frac{1}{x^2}$  for  $x > 0$ , together with a set of  $n$  rectangles of unit width, starting at  $x = 1$ .

(i) By considering the areas of these rectangles, explain why

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} > \int_1^{n+1} \frac{1}{x^2} dx. \quad [2]$$

(ii) By considering the areas of another set of rectangles, explain why

$$\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots + \frac{1}{n^2} < \int_1^n \frac{1}{x^2} dx. \quad [3]$$

(iii) Hence show that

$$1 - \frac{1}{n+1} < \sum_{r=1}^n \frac{1}{r^2} < 2 - \frac{1}{n}. \quad [4]$$

(iv) Hence give bounds between which  $\sum_{r=1}^{\infty} \frac{1}{r^2}$  lies. [2]

7 (i) Using the definitions of hyperbolic functions in terms of exponentials, prove that

$$\cosh x \cosh y - \sinh x \sinh y = \cosh(x - y). \quad [4]$$

(ii) Given that  $\cosh x \cosh y = 9$  and  $\sinh x \sinh y = 8$ , show that  $x = y$ . [2]

(iii) Hence find the values of  $x$  and  $y$  which satisfy the equations given in part (ii), giving the answers in logarithmic form. [4]

## 4

8 The iteration  $x_{n+1} = \frac{1}{(x_n + 2)^2}$ , with  $x_1 = 0.3$ , is to be used to find the real root,  $\alpha$ , of the equation  $x(x + 2)^2 = 1$ .

(i) Find the value of  $\alpha$ , correct to 4 decimal places. You should show the result of each step of the iteration process. [4]

(ii) Given that  $f(x) = \frac{1}{(x + 2)^2}$ , show that  $f'(\alpha) \neq 0$ . [2]

(iii) The difference,  $\delta_r$ , between successive approximations is given by  $\delta_r = x_{r+1} - x_r$ . Find  $\delta_3$ . [1]

(iv) Given that  $\delta_{r+1} \approx f'(\alpha)\delta_r$ , find an estimate for  $\delta_{10}$ . [3]

9 It is given that the equation of a curve is

$$y = \frac{x^2 - 2ax}{x - a},$$

where  $a$  is a positive constant.

(i) Find the equations of the asymptotes of the curve. [4]

(ii) Show that  $y$  takes all real values. [4]

(iii) Sketch the curve  $y = \frac{x^2 - 2ax}{x - a}$ . [3]

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