4723 Core Mathematics 3

1 (i)	Show correct process for composition of functions	**M1**	numerical or algebraic; the right way round
	Obtain (–3 and hence) –23	**A1 2**	
(ii)	<u>Either:</u> State or imply $x^3 + 4 = 12$	**B1**	
	Attempt solution of equation involving x^3	**M1**	as far as $x = \ldots$
	Obtain 2	**A1 3**	and no other value
	<u>Or:</u> Attempt expression for f^{-1}	**M1**	involving x or y; involving cube root
	Obtain $\sqrt[3]{x-4}$ or $\sqrt[3]{y-4}$	**A1**	
	Obtain 2	**A1 (3)**	and no other value

2 (i)	Obtain correct first iterate 2.864	**B1**	or greater accuracy 2.864327…; condone 2 dp here and in working
	Carry out correct iteration process	**M1**	to find at least 3 iterates in all
	Obtain 2.877	**A1 3**	after at least 4 steps; answer required to exactly 3 dp

$$[3 \rightarrow 2.864327 \rightarrow 2.878042 \rightarrow 2.876661 \rightarrow 2.876800]$$

(ii)	State or imply $x = \sqrt[3]{31 - \tfrac{5}{2}x}$	**B1**	
	Attempt rearrangement of equation in x	**M1**	involving cubing and grouping non-zero terms on LHS
	Obtain equation $2x^3 + 5x - 62 = 0$	**A1 3**	or equiv with integers

3 (a)	State correct equation involving $\cos\tfrac{1}{2}\alpha$	**B1**	such as $\cos\tfrac{1}{2}\alpha = \tfrac{1}{4}$ or $\dfrac{1}{\cos\tfrac{1}{2}\alpha} = 4$ or …
	Attempt to find value of α	**M1**	using correct order for the steps
	Obtain 151	**A1 3**	or greater accuracy; and no other values between 0 and 180
(b)	State or imply $\cot\beta = \dfrac{1}{\tan\beta}$	**B1**	
	Rearrange to the form $\tan\beta = k$	**M1**	or equiv involving $\sin\beta$ only or $\cos\beta$ only; allow missing $\pm$
	Obtain 69.3	**A1**	
	Obtain 111	**A1 4**	or greater accuracy; and no others between 0 and 180

4 (i)	Obtain derivative of form $kh^5(h^6 + 16)^n$	**M1**	any constant k; any $n < \tfrac{1}{2}$; allow if – 4 term retained
	Obtain correct $3h^5(h^6 + 16)^{-\frac{1}{2}}$	**A1**	or (unsimplified) equiv; no –4 now
	Substitute to obtain 10.7	**A1 3**	or greater accuracy or exact equiv
(ii)	Attempt multn or divn using 8 and answer from (i) **M1**		
	Attempt 8 divided by answer from (i)	**M1**	
	Obtain 0.75	**A1√ 3**	or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

5 (a) Obtain integral of form $k(3x+7)^{10}$ — **M1** any constant k

Obtain (unsimplified) $\frac{1}{10} \times \frac{1}{3}(3x+7)^{10}$ — **A1** or equiv

Obtain (simplified) $\frac{1}{30}(3x+7)^{10} + c$ — **A1 3**

(b) State $\int \pi \left(\frac{1}{2\sqrt{x}}\right)^2 dx$ — **B1** or equiv involving x; condone no dx

Integrate to obtain $k \ln x$ — **M1** any constant k involving π or not; or equiv such as $k \ln 4x$ or $k \ln 2x$

Obtain $\frac{1}{4}\pi \ln x$ or $\frac{1}{4}\ln x$ or $\frac{1}{4}\pi \ln 4x$ or $\frac{1}{4}\ln 4x$ **A1**

Show use of the $\log a - \log b$ property — **M1** not dependent on earlier marks

Obtain $\frac{1}{4}\pi \ln 2$ — **A1 5** or similarly simplified equiv

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6 (i) <u>Either</u>: Refer to translation and reflection — **B1** in either order; allow clear equivs

State translation by 1 in negative x-direction — **B1** or equiv but now using correct terminology

State reflection in x-axis — **B1 3** using correct terminology

<u>Or</u>: Refer to translation and reflection — **B1** in either order; allow clear equivs

State reflection in y-axis — **B1**

State translation by 1 in positive x-direction — **B1 (3)** with order reflection then translation clearly intended

(ii) Show sketch with attempt at reflection of 'negative' part in x-axis — **M1** and curve for 0<x<1 unchanged

Show (more or less) correct sketch — **A1 2** with correct curvature

(iii) Attempt correct process for finding at least one value — **M1** as far as $x = \ldots$; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$

Obtain $1 - \frac{1}{2}\sqrt{3}$ — **A1** or exact equiv

Obtain $1 + \frac{1}{2}\sqrt{3}$ — **A1 3** or exact equiv; give **A1A0** if extra incorrect solution(s) provided

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7 (i) Attempt use of product rule for $x\mathrm{e}^{2x}$ — **M1** obtaining $\ldots + \ldots$

Obtain $\mathrm{e}^{2x} + 2x\mathrm{e}^{2x}$ — **A1** or equiv; maybe within QR attempt

Attempt use of quotient rule — **M1** with or without product rule

Obtain unsimplified $\dfrac{(x+k)(\mathrm{e}^{2x}+2x\mathrm{e}^{2x}) - x\mathrm{e}^{2x}}{(x+k)^2}$ — **A1**

Obtain $\dfrac{\mathrm{e}^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$ — **A1 5** **AG**; necessary detail required

(ii) Attempt use of discriminant — **M1** or equiv

Obtain $4k^2 - 8k = 0$ or equiv and hence $k = 2$ — **A1**

Attempt solution of $2x^2 + 2kx + k = 0$ — **M1** using their numerical value of k or solving in terms of k using correct formula

Obtain $x = -1$ — **A1**

Obtain $-\mathrm{e}^{-2}$ — **A1 5** or exact equiv

8 (i) State or imply $h = 1$ **B1**

Attempt calculation involving attempts at y values **M1** addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 y values

Obtain $a(1 + 4\times2 + 2\times4 + 4\times8 + 2\times16 + 4\times32 + 64)$**A1** any constant a

Obtain 91 **A1 4**

(ii) State $e^{x\ln 2}$ or $k = \ln 2$ **B1** allow decimal equiv such as $e^{0.69x}$

Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$ **M1** any constant k or in terms of general k

Obtain $\frac{1}{\ln 2}(e^{6\ln 2} - e^0)$ **A1** or exact equiv

Simplify to obtain $\frac{63}{\ln 2}$ **A1 4** allow if simplification in part (iii)

(iii) Equate answers to (i) and (ii) **M1** provided ln 2 involved other than in power of e

Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$ **A1 2** **AG**; necessary correct detail required

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9 (i) State at least one of $\cos\theta\cos 60 - \sin\theta\sin 60$
and $\cos\theta\cos 30 - \sin\theta\sin 30$ **B1**

Attempt complete multiplication of identities of form $\pm\cos\cos \pm \sin\sin$ **M1** with values $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}$ involved

Use $\cos^2\theta + \sin^2\theta = 1$ and $2\sin\theta\cos\theta = \sin 2\theta$ **M1**

Obtain $\sqrt{3} - 2\sin 2\theta$ **A1 4** **AG**; necessary detail required

(ii) Attempt use of 22.5 in right-hand side **M1**

Obtain $\sqrt{3} - \sqrt{2}$ **A1 2** or exact equiv

(iii) Obtain 10.7 **B1** or greater accuracy; allow ± 0.1

Attempt correct process to find two angles **M1** from values of 2θ between 0 and 180

Obtain 79.3 **A1 3** or greater accuracy and no others between 0 and 90; allow ± 0.1

(iv) Indicate or imply that critical values of $\sin 2\theta$ are -1 and 1 **M1**

Obtain both of $k > \sqrt{3} + 2$, $k < \sqrt{3} - 2$ **A1** condoning decimal equivs, $\leq \geq$ signs

Obtain complete correct solution **A1 3** now with exact values and unambiguously stated