OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 4

4724

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find the quotient and the remainder when $x^4 + 11x^3 + 28x^2 + 3x + 1$ is divided by $x^2 + 5x + 2$. **[4]**

2 Points A, B and C have position vectors $-5\mathbf{i} - 10\mathbf{j} + 12\mathbf{k}$, $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$ respectively, where p is a constant.

(i) Given that angle $ABC = 90°$, find the value of p. **[4]**

(ii) Given instead that ABC is a straight line, find the value of p. **[2]**

3 By expressing $\cos 2x$ in terms of $\cos x$, find the exact value of $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \frac{\cos 2x}{\cos^2 x}\,\mathrm{d}x$. **[5]**

4 Use the substitution $u = 2 + \ln t$ to find the exact value of

$$\int_1^{\mathrm{e}} \frac{1}{t(2 + \ln t)^2}\,\mathrm{d}t.$$ **[6]**

5 **(i)** Expand $(1 + x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[2]**

(ii) **(a)** Hence, or otherwise, expand $(8 + 16x)^{\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2. **[4]**

(b) State the set of values of x for which the expansion in part **(ii) (a)** is valid. **[1]**

6 A curve has parametric equations

$$x = 9t - \ln(9t), \quad y = t^3 - \ln(t^3).$$

Show that there is only one value of t for which $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3$ and state that value. **[6]**

7 Find the equation of the normal to the curve $x^3 + 2x^2y = y^3 + 15$ at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[8]**

8 **(i)** State the derivative of $\mathrm{e}^{\cos x}$. **[1]**

(ii) Hence use integration by parts to find the exact value of

$$\int_0^{\frac{1}{2}\pi} \cos x \sin x\, \mathrm{e}^{\cos x}\,\mathrm{d}x.$$ **[6]**

9 The equation of a straight line l is $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. O is the origin.

(i) The point P on l is given by $t = 1$. Calculate the acute angle between OP and l. **[4]**

(ii) Find the position vector of the point Q on l such that OQ is perpendicular to l. **[4]**

(iii) Find the length of OQ. **[2]**

10 **(i)** Express $\dfrac{1}{(3-x)(6-x)}$ in partial fractions. **[2]**

(ii) In a chemical reaction, the amount x grams of a substance at time t seconds is related to the rate at which x is changing by the equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(3-x)(6-x),$$

where k is a constant. When $t = 0$, $x = 0$ and when $t = 1$, $x = 1$.

(a) Show that $k = \frac{1}{3}\ln\frac{5}{4}$. **[7]**

(b) Find the value of x when $t = 2$. **[4]**

