



ADVANCED GCE
MATHEMATICS
Probability & Statistics 3

4734

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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- 1 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{5} & -a \leq x < 0, \\ \frac{2}{5}e^{-2x} & x \geq 0. \end{cases}$$

Find

- (i) the value of the constant a , [3]
 (ii) $E(X)$. [5]

- 2 The amount of tomato juice, X ml, dispensed into cartons of a particular brand has a normal distribution with mean 504 and standard deviation 3. The juice is sold in packs of 4 cartons, filled independently. The total amount of juice in one pack is Y ml.

- (i) Find $P(Y < 2000)$. [4]

The random variable V is defined as $Y - 4X$.

- (ii) Find $E(V)$ and $\text{Var}(V)$. [3]
 (iii) What is the probability that the amount of juice in a randomly chosen pack is more than 4 times the amount of juice in a randomly chosen carton? [1]

- 3 It is given that X_1 and X_2 are independent random variables with $X_1 \sim N(\mu_1, 2.47)$ and $X_2 \sim N(\mu_2, 4.23)$. Random samples of n_1 observations of X_1 and n_2 observations of X_2 are taken. The sample means are denoted by \bar{X}_1 and \bar{X}_2 .

- (i) State the distribution of $\bar{X}_1 - \bar{X}_2$, giving its parameters. [3]

For two particular samples, $n_1 = 5$, $\Sigma x_1 = 48.25$, $n_2 = 10$ and $\Sigma x_2 = 72.30$.

- (ii) Test at the 2% significance level whether μ_1 differs from μ_2 . [6]

A student stated that because of the Central Limit Theorem the sample means will have normal distributions so it is unnecessary for X_1 and X_2 to have normal distributions.

- (iii) Comment on the student's statement. [1]

- 4 The continuous random variable V has (cumulative) distribution function given by

$$F(v) = \begin{cases} 0 & v < 1, \\ 1 - \frac{8}{(1+v)^3} & v \geq 1. \end{cases}$$

The random variable Y is given by $Y = \frac{1}{1+V}$.

- (i) Show that the (cumulative) distribution function of Y is $8y^3$, over an interval to be stated, and find the probability density function of Y . [7]

- (ii) Find $E\left(\frac{1}{Y^2}\right)$. [2]

3

- 5 Each of a random sample of 200 steel bars taken from a production line was examined and 27 were found to be faulty.

(i) Find an approximate 90% confidence interval for the proportion of faulty bars produced. [4]

A change in the production method was introduced which, it was claimed, would reduce the proportion of faulty bars. After the change, each of a further random sample of 100 bars was examined and 8 were found to be faulty.

(ii) Test the claim, at the 10% significance level. [7]

- 6 The deterioration of a certain drug over time was investigated as follows. The drug strength was measured in each of a random sample of 8 bottles containing the drug. These were stored for two years and the strengths were then re-measured. The original and final strengths, in suitable units, are shown in the following table.

| | | | | | | | | |
|-------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Bottle | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Original strength | 8.7 | 9.4 | 9.2 | 8.9 | 9.6 | 8.2 | 9.9 | 8.8 |
| Final strength | 8.1 | 9.0 | 9.0 | 8.8 | 9.3 | 8.0 | 9.5 | 8.5 |

(i) Stating any required assumption, test at the 5% significance level whether the mean strength has decreased by more than 0.2 over the two years. [9]

(ii) Calculate a 95% confidence interval for the mean reduction in strength over the two years. [3]

- 7 A chef wished to ascertain her customers' preference for certain vegetables. She asked a random sample of 120 customers for their preferred vegetable from asparagus, broad beans and cauliflower. The responses, classified according to the gender of the customer, are shown in the table.

| | | | |
|-------------------|-----------|-------------|-------------|
| | Asparagus | Broad beans | Cauliflower |
| Female preference | 31 | 9 | 25 |
| Male preference | 17 | 21 | 17 |

(i) Test, at the 5% significance level, whether vegetable preference and gender are independent. [8]

(ii) Determine whether, at the 10% significance level, the vegetables are equally preferred. [6]

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