# Mark Scheme 4723
# June 2007

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|---|---|---|---|---|---|
| **1** | **(i)** | Attempt use of product rule | M1 | | |
| | | Obtain $3x^2(x+1)^5 + 5x^3(x+1)^4$ | A1 | **2** | or equiv |
| | | [<u>Or</u>: (following complete expansion and differentiation term by term) | | | |
| | | Obtain $8x^7 + 35x^6 + 60x^5 + 50x^4 + 20x^3 + 3x^2$ | B2 | | allow B1 if one term incorrect] |
| | **(ii)** | Obtain derivative of form $kx^3(3x^4+1)^n$ | M1 | | any constants $k$ and $n$ |
| | | Obtain derivative of form $kx^3(3x^4+1)^{-\frac{1}{2}}$ | M1 | | |
| | | Obtain correct $6x^3(3x^4+1)^{-\frac{1}{2}}$ | A1 | **3** | or (unsimplified) equiv |
| **2** | | Identify critical value $x = 2$ | B1 | | |
| | | Attempt process for determining both critical values | M1 | | |
| | | Obtain $\frac{1}{3}$ and 2 | A1 | | |
| | | Attempt process for solving inequality | M1 | | table, sketch …; implied by plausible answer |
| | | Obtain $\frac{1}{3} < x < 2$ | A1 | **5** | |
| **3** | **(i)** | Attempt correct process for composition | M1 | | numerical or algebraic |
| | | Obtain (16 and hence) 7 | A1 | **2** | |
| | **(ii)** | Attempt correct process for finding inverse | M1 | | maybe in terms of $y$ so far |
| | | Obtain $(x-3)^2$ | A1 | **2** | or equiv; in terms of $x$, not $y$ |
| | **(iii)** | Sketch (more or less) correct $y = \mathrm{f}(x)$ | B1 | | with 3 indicated or clearly implied on $y$-axis, correct curvature, no maximum point |
| | | Sketch (more or less) correct $y = \mathrm{f}^{-1}(x)$ | B1 | | right hand half of parabola only |
| | | State reflection in line $y = x$ | B1 | **3** | or (explicit) equiv; independent of earlier marks |
| **4** | **(i)** | Obtain integral of form $k(2x+1)^{\frac{4}{3}}$ | M1 | | or equiv using substitution; any constant $k$ |
| | | Obtain correct $\frac{3}{8}(2x+1)^{\frac{4}{3}}$ | A1 | | or equiv |
| | | Substitute limits in expression of form $(2x+1)^n$ and subtract the correct way round | M1 | | using adjusted limits if subn used |
| | | Obtain 30 | A1 | **4** | |
| | **(ii)** | Attempt evaluation of $k(y_0 + 4y_1 + y_2)$ | M1 | | any constant $k$ |
| | | Identify $k$ as $\frac{1}{3}\times 6.5$ | A1 | | |
| | | Obtain 29.6 | A1 | **3** | or greater accuracy (29.554566…) |
| | | [SR: (using Simpson's rule with 4 strips) | | | |
| | | Obtain $\frac{1}{3}\times 3.25(1 + 4\times\sqrt[3]{7.5} + 2\times\sqrt[3]{14} + 4\times\sqrt[3]{20.5} + 3)$ and hence 29.9 | B1 | | or greater accuracy (29.897…)] |

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| **5 (i)** | State $e^{-0.04t} = 0.5$ | B1 | | or equiv |
| | Attempt solution of equation of form $e^{-0.04t} = k$ | M1 | | using sound process; maybe implied |
| | Obtain 17 | A1 | **3** | or greater accuracy (17.328…) |
| **(ii)** | Differentiate to obtain form $k\,e^{-0.04t}$ | *M1 | | constant $k$ different from 240 |
| | Obtain (±) $9.6e^{-0.04t}$ | A1 | | or (unsimplified) equiv |
| | Equate attempt at first derivative to (±) 2.1 and attempt solution | M1 | | dep *M; method maybe implied |
| | Obtain 38 | A1 | **4** | or greater accuracy (37.9956…) |
| **6 (i)** | Obtain integral of form $k_1 e^{2x} + k_2 x^2$ | M1 | | any non-zero constants $k_1, k_2$ |
| | Obtain correct $3e^{2x} + \frac{1}{2}x^2$ | A1 | | |
| | Obtain $3e^{2a} + \frac{1}{2}a^2 - 3$ | A1 | | |
| | Equate definite integral to 42 and attempt rearrangement | M1 | | using sound processes |
| | Confirm $a = \frac{1}{2}\ln(15 - \frac{1}{6}a^2)$ | A1 | **5** | AG; necessary detail required |
| **(ii)** | Obtain correct first iterate 1.348… | B1 | | |
| | Attempt correct process to find at least 2 iterates | M1 | | |
| | Obtain at least 3 correct iterates | A1 | | |
| | Obtain 1.344 | A1 | **4** | answer required to exactly 3 d.p.; allow recovery after error |
| | [$1 \rightarrow 1.34844 \rightarrow 1.34382 \rightarrow 1.34389$] | | | |
| **7 (i)** | Show correct general shape (alternating above and below $x$-axis) | M1 | | with no branch reaching $x$-axis |
| | Draw (more or less) correct sketch | A1 | **2** | with at least one of 1 and –1 indicated or clearly implied |
| **(ii)** | Attempt solution of $\cos x = \frac{1}{3}$ | M1 | | maybe implied; or equiv |
| | Obtain 1.23 or $0.392\pi$ | A1 | | or greater accuracy |
| | Obtain 5.05 or $1.61\pi$ | A1 | **3** | or greater accuracy and no others within $0 \le x \le 2\pi$; penalise answer(s) to 2sf only once |
| **(iii)** | <u>Either</u>: Obtain equation of form $\tan\theta = k$ M1 | any constant $k$; maybe implied | | |
| | Obtain $\tan\theta = 5$ | A1 | | |
| | Obtain two values only of form $\theta,\ \theta + \pi$ | M1 | | within $0 \le x \le 2\pi$; allow degrees at this stage |
| | Obtain 1.37 and 4.51 (or $0.437\pi$ and $1.44\pi$) | A1 | **4** | allow ±1 in third sig fig; or greater accuracy |
| | <u>Or</u>: (for methods which involve squaring,etc.) | | | |
| | Attempt to obtain eqn in one trig ratio | M1 | | |
| | Obtain correct value | A1 | | $\tan^2\theta = 25, \cos^2\theta = \frac{1}{26}$, … |
| | Attempt solution at least to find one value in first quadrant and one value in third | M1 | | |
| | Obtain 1.37 and 4.51 (or equivs as above) | A1 | | ignoring values in second and fourth quadrants |

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| 8 | (i) | Attempt use of quotient rule | M1 | | allow for numerator 'wrong way round'; or equiv |
| | | Obtain $\dfrac{(4\ln x+3)\frac{4}{x}-(4\ln x-3)\frac{4}{x}}{(4\ln x+3)^2}$ | A1 | | or equiv |
| | | Confirm $\dfrac{24}{x(4\ln x+3)^2}$ | A1 | **3** | AG; necessary detail required |
| | (ii) | Identify $\ln x=\frac{3}{4}$ | B1 | | or equiv |
| | | State or imply $x=e^{\frac{3}{4}}$ | B1 | | |
| | | Substitute $e^k$ completely in expression for derivative | M1 | | and deal with $\ln e^k$ term |
| | | Obtain $\frac{2}{3}e^{-\frac{3}{4}}$ | A1 | **4** | or exact (single term) equiv |
| | (iii) | State or imply $\int \dfrac{4\pi}{x(4\ln x+3)^2}\,dx$ | B1 | | |
| | | Obtain integral of form $k\dfrac{4\ln x-3}{4\ln x+3}$ or $k(4\ln x+3)^{-1}$ | *M1 | | any constant $k$ |
| | | Substitute both limits and subtract right way round | M1 | | dep *M |
| | | Obtain $\frac{4}{21}\pi$ | A1 | **4** | or exact equiv |
| 9 | (i) | Attempt use of either of $\tan(A\pm B)$ identities | M1 | | |
| | | Substitute $\tan 60°=\sqrt{3}$ or $\tan^2 60°=3$ | B1 | | |
| | | Obtain $\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}\times\dfrac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}$ | A1 | | or equiv (perhaps with tan 60° still involved) |
| | | Obtain $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}$ | A1 | **4** | AG |
| | (ii) | Use $\sec^2\theta=1+\tan^2\theta$ | B1 | | |
| | | Attempt rearrangement and simplification of equation involving $\tan^2\theta$ | M1 | | or equiv involving $\sec\theta$ |
| | | Obtain $\tan^4\theta=\frac{1}{3}$ | A1 | | or equiv $\sec^2\theta=1.57735...$ |
| | | Obtain 37.2 | A1 | | or greater accuracy |
| | | Obtain 142.8 | A1 | **5** | or greater accuracy; and no others between 0 and 180 |
| | (iii) | Attempt rearrangement of $\dfrac{\tan^2\theta-3}{1-3\tan^2\theta}=k^2$ to form $\tan^2\theta=\dfrac{f(k)}{g(k)}$ | M1 | | |
| | | Obtain $\tan^2\theta=\dfrac{k^2+3}{1+3k^2}$ | A1 | | |
| | | Observe that RHS is positive for all $k$, giving one value in each quadrant | A1 | **3** | or convincing equiv |