

4723

Mark Scheme

June 2010

- 1 (i) Attempt use of product rule
Obtain $3x^2e^{2x} + 2x^3e^{2x}$ M1 producing ... + ... form
A1 2 or equiv
-
- (ii) Attempt use of chain rule to produce $\frac{kx}{3+2x^2}$ form M1 any constant k
Obtain $\frac{4x}{3+2x^2}$ A1 2
-
- (iii) Attempt use of quotient rule M1 or equiv; condone u/v confusions
Obtain $\frac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1} - 2x(2x+1)^{-2}$ A1 2 or (unsimplified) equiv
- [If ... + c included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise ignore any inclusion of ... + c .]

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- 2 (i) Obtain one of $\pm \ln(\pm x \pm 4)$ M1
Obtain correct equation $y = -\ln(x-4)$ A1 2 or equiv; condone use of modulus signs instead of brackets
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- (ii) State, in any order, S, S and T M1 or equiv such as S^2 , T or 2S, T
State T, then S, then S A1 2 or equiv (note that S, S, T^9 and S, T^3 , S are alternative correct answers)

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- 3 (i) Use $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ B1
Attempt to express equation in terms of $\sin \theta$ M1 using $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ or equiv
Obtain or clearly imply $6 \sin^2 \theta - 11 \sin \theta - 10 = 0$ A1 3 or $-6 \sin^2 \theta + 11 \sin \theta + 10 = 0$
-
- (ii) Attempt solution to obtain at least one value of $\sin \theta$ M1 should be $s = -\frac{2}{3}, \frac{5}{2}$
Obtain -41.8 A1 allow -42 or greater accuracy
Obtain -138 A1 3 or greater accuracy; and no others between -180 and 180
- [Answer(s) only: award 0 out of 3.]

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4	<p>(i) <u>Either</u>: Integrate to obtain $k \ln x$ B1 Use at least one relevant logarithm property M1 Obtain $k \ln 3 = \ln 81$ and hence $k = 4$ A1 3 AG; accurate work required</p> <p><u>Or 1</u>: (where solution involves no use of a logarithm property) Integrate to obtain $k \ln x$ B1 Obtain correct explicit expression for k and conclude $k = 4$ with no error seen B2 3 AG; e.g. $k = \frac{\ln 81}{\ln 6 - \ln 2} = 4$</p> <p><u>Or 2</u>: (where solution involves verification of result by initial substitution of 4 for k) Integrate to obtain $4 \ln x$ B1 Use at least one relevant logarithm property M1 Obtain $\ln 81$ legitimately with no error seen A1 3 AG; accurate work required</p>

(ii)	<p>State volume involves $\int \pi \left(\frac{4}{x}\right)^2 dx$ B1 possibly implied</p> <p>Obtain integral of form $k_1 x^{-1}$ M1 any constant k_1 including π or not</p> <p>Use correct process for finding volume produced from S M1 $\int (k_2 2^2 - k_3 y^2) dx$, including π or not with correct limits indicated; or equiv</p> <p>Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A1 4 or exact equiv</p> <p style="text-align: right;">7</p>

5	<p>(i) Attempt process for finding both critical values M1 squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities</p> <p>Obtain -4 A1</p> <p>Obtain $\frac{2}{3}$ A1</p> <p>Attempt process for solving inequality M1 table, sketch, ...; needs two critical values; implied by plausible answer</p> <p>Obtain $-4 \leq x \leq \frac{2}{3}$ A1 5 with \leq and not $<$</p>

(ii)	<p>Use correct process to find value of $x + 2$ using any value M1 ... whether part of answer to (i) or not</p> <p>Obtain $2\frac{2}{3}$ or $\frac{8}{3}$ A1 2 dependent on 5 marks awarded in part (i)</p> <p style="text-align: right;">7</p>

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6	<p>(i) Attempt calculations involving 1.0 and 1.1 Obtain -0.57 and 0.76</p> <p>Refer to sign change (or equiv for rearranged eqn)</p>	<p>M1 using radians A1 or values to 1 dp (rounded or truncated); or equivs (where eqn rearranged) A1 3 AG; following correct work only</p>
<hr/>		
(ii)	<p>Obtain correct first iterate Carry out iteration process Obtain at least 3 correct iterates Obtain 1.05083</p> <p>[1 \rightarrow 1.047198 \rightarrow 1.050571 \rightarrow 1.050809 \rightarrow 1.050826 \rightarrow 1.050827; 1.05 \rightarrow 1.050769 \rightarrow 1.050823 \rightarrow 1.050827 \rightarrow 1.050827; 1.1 \rightarrow 1.054268 \rightarrow 1.051070 \rightarrow 1.050844 \rightarrow 1.050829 \rightarrow 1.050827]</p>	<p>B1 using value x_1 such that $1.0 \leq x_1 \leq 1.1$ M1 obtaining at least 3 iterates in all so far A1 showing at least 3 dp A1 4 answer required to exactly 5 d.p.</p>
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(iii)	<p>State or imply $\sec^2 2x = 1 + \tan^2 2x$ Relate to earlier equation</p> <p>Deduce $2x = 1.05083$ and hence 0.525</p> <p>[SC: Rearrange to obtain $x = \frac{1}{2} \cos^{-1}(2x+3)^{-\frac{1}{2}}$ Use iterative process to obtain 0.525</p>	<p>B1 M1 by halving or doubling answer to (ii) or carrying out equivalent iteration process A1 3 following their answer to (ii); or greater accuracy B1 B1 2 or greater accuracy]</p>
10		
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7	<p>Differentiate to obtain $k_1(3x-1)^3$ Obtain correct $12(3x-1)^3$ Substitute 1 to obtain 96 Attempt to find x-coordinate of Q Obtain $\frac{5}{6}$</p> <p>Integrate to obtain $k_2(3x-1)^5$ Obtain correct $\frac{1}{15}(3x-1)^5$ Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$ Attempt to find shaded area by correct process Obtain $(\frac{32}{15} - \frac{1}{2} \times \frac{1}{6} \times 16)$ and hence $\frac{4}{5}$</p>	<p>M1 any constant k_1 A1 or (unsimplified) equiv A1 M1 using tangent with $y = 0$ or using gradient A1 or exact equiv M1 any constant k_2 A1 or (unsimplified) equiv A1 M1 integral – triangle or equiv A1 or equiv</p>
10		
<hr/>		
8	<p>(i) Obtain $R = 3\sqrt{2}$ or $R = \sqrt{18}$ or $R = 4.24$ Attempt to find value of α Obtain $\frac{1}{4}\pi$ or 0.785</p>	<p>B1 or equiv M1 condone sin/cos muddles and degrees A1 3 in radians now</p>
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(ii) a	<p>Equate $x - \alpha$ to $\frac{1}{2}\pi$ or attempt solution of $3\cos x + 3\sin x = 0$ Obtain $\frac{3}{4}\pi$</p>	<p>M1 condone degrees here A1 2 or ..., $-\frac{5}{4}\pi, -\frac{1}{4}\pi, \frac{7}{4}\pi, \dots$; in radians now</p>
<hr/>		
b	<p>Attempt correct process to find value of $3x - \alpha$ Obtain at least one correct exact value of $3x - \alpha$ Attempt at least one positive value of x Obtain $\frac{1}{36}\pi$</p>	<p>*M1 with attempt at rearranging $T(3x) = \frac{8}{9}\sqrt{6}$ A1 $\pm\frac{1}{6}\pi, \pm\frac{11}{6}\pi, \dots$ M1 dep *M A1 4</p>
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<p>9 (i) Attempt to find x-coord of staty point or complete square Obtain $(\frac{3}{2}, -9)$ or $4(x - \frac{3}{2})^2 - 9$ or -9 State $f(x) \geq -9$</p>	<p>M1 A1 or equiv A1 3 using any notation; with \geq</p>
<hr style="border-top: 1px dashed black;"/>	
<p>(ii) Make one correct (perhaps general) relevant statement Conclude with correct evidence related to this f</p>	<p>B1 not 1 -1, f is many-one, ... ; maybe implied if attempt is specific to this f B1 2 AG; (more or less) correct sketch; correct relevant calculations, ...</p>
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<p>(iii) <u>Either</u>: Attempt to find expression for g^{-1} Obtain $\frac{1}{a}(x-b)$ Compare $\frac{1}{a}(x-b)$ and $ax+b$ Obtain at least $-\frac{b}{a} = b$ and hence $a = -1$ [SC1: first two steps as above, then substitute $a = -1$: max possible M1A1B1] [SC2: substitute $a = -1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 2] <u>Or</u>: State or imply that $y = g^{-1}(x)$ is reflection of $y = g(x)$ in line $y = x$ State that line unchanged by this reflection is perpendicular to $y = x$ Conclude that a is -1</p>	<p>*M1 or equiv A1 or equiv M1 dep *M; by equating either coefficients of x or constant terms (or both); or substituting two non-zero values of x and solving eqns for a A1 4 AG; necessary detail required; or equiv M1 Obtain $-x+b$ and conclude A1 2 B1 M2 A1 4</p>
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<p>(iv) State or imply that $gf(x) = -(4x^2 - 12x) + b$ Attempt use of discriminant or relate to range of f Obtain $64 + 16b < 0$ or $9 + b < 5$ Obtain $b < -4$</p>	<p>B1 M1 or equiv A1 or equiv A1 4 13</p>