

# 4734 Probability & Statistics 3

1(i)	$s^2 = 0.00356/80 + 0.00340/100$ $= 7.85 \times 10^{-5}$	M1 A1 2	Sum of variances Or pooled, giving $7.81 \times 10^{-5}$
(ii)	----- $(1.36 - 1.24) \pm z s$ $z = 1.96$ $(0.103, 0.137)$ -----	M1 B1 A1 3	Must be $s$ , accept $t$
(iii)	Not necessary since sample sizes are large	B1 1 (6)	Or equivalent. Nothing wrong
2(i)	Use $\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$ $\bar{x} = 337.5 / 20$ $z = 2.326$ $(14.9, 18.9)$	M1  B1 B1 A1 4	3 or 4 SF
(ii)	----- $1 - 0.98^3$ 0.0588 -----	M1 A1 2	Use B(3,0.02) or B(3,0.98) for M.
(iii)	Unbiased estimate of $\sigma^2$ required $t$ - distribution used to obtain CV	B1  B1 2 (8)	
3(i)	$H_0: p_W = p_N, H_1: p_W > p_N$ Pooled $\hat{p} = \frac{71+73}{80+90} \quad (= \frac{144}{170})$  $s^2 = (144/170)(26/170)(1/80+1/90)$ $z = (71/80 - 73/90)/s$ $= 1.381$ $1.381 < 1.645$ Do not reject $H_0$ , there is insufficient evidence that the proportion of on-time Western trains exceeds the proportion of on-time Northern trains	B1  B1  B1 M1 A1  M1  A1 7	For both hypotheses. Or $\pi$ . SR: from $p_1q_1/n_1 + p_2q_2/n_2 = 0.00295$ $z = 1.406$ B1M1A1M1A1 Max 5/7  If no explicit comparison and correct conclusion then M1A0. Or use P-value or CR In context, not too assertive
(ii)	----- $s^2 = 71 \times 9/80^3 + 73 \times 17/90^3$ $= 0.00295$	M1 A1 2 (9)	AEF Allow one error Accept 0.0029
4(i)	Use $L - S_1 - S_2$ $\mu = 0.7$ $\sigma^2 = 0.58^2 + 0.31^2 + 0.31^2$ $= 0.5286$ $(1 - 0.7)/\sigma$ 0.340	M1 B1 M1 A1 M1 A1 6	Or equivalent, or implied  May be implied later Correct numerator
(ii)	----- Use $L - 2S$ with $\mu = 0.7$ $\sigma^2 = 0.58^2 + 4(0.31)^2$ $- 0.7/\sigma$ $- 0.824(5)$ 0.2048	M*1 B1 Dep*M1 A1 A1 5 (11)	M0 if as (i) unless correct Accept + 0.205 (3SF)

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<p>5(i)</p>	<p>Population of differences is normal  <math>H_0: \mu_A = \mu_B</math>, <math>H_1: \mu_A &lt; \mu_B</math> where <math>\mu_A</math> and <math>\mu_B</math> denote the population means  <math>\bar{x}_D = 3.222</math>  <math>s_D = 5.019</math>   <math>t = 3.222/(5.019/3)</math>  <math>= 1.926</math>  <math>CV = 1.860</math>  <math>1.926 &gt; 1.860</math>                  Reject <math>H_0</math>, there is evidence that brand A takes less time than brand B</p>	<p>B1                  B1                   B1                  M1A1                   M1                  A1                  B1                  M1                   A1 <b>10</b></p>	<p><i>Not "independent"</i>                  Or <math>\mu_D = 0, \mu_D &gt; 0</math>                   From formula, or B2 from calculator                   Accept 1.93. M1A0 if <math>t = -1.926</math></p>												
<p>(ii)</p>	<p>One valid reason</p>	<p>B1 <b>1 (11)</b></p>	<p>Data are clearly paired                  Data not independent</p>												
<p>6(i)</p>	<p><math>37 \times 58 / 120</math>  <math>17.883\dots</math>, 17.88 AG</p>	<p>M1                  A1 <b>2</b></p>	<p>Or equivalent</p>												
<p>(ii)</p>	<p><math>H_0</math>: Gender and shade are independent  <math>(H_1</math>:--are not independent)  <math>3.02^2(14.02^{-1} + 14.98^{-1}) +</math>  <math>6.12^2(17.88^{-1} + 19.12^{-1}) +</math>  <math>3.1^2(26.1^{-1} + 27.9^{-1})</math>  <math>= 6.03</math>                  EITHER: CV 5.991  <math>6.03 &gt; 5.991</math>, reject <math>H_0</math> and accept that gender and shade are not independent                  OR: <math>P(\chi^2 &gt; 6.03) = 0.049</math>  <math>&lt; 0.05</math>, reject <math>H_0</math> and accept that gender and shade are not independent</p>	<p>B1                   M1                  A1                   A1                  B1                  M1                  A1√ <b>7</b>                  B1                  M1                  A1√</p>	<p>At least two correct                  All correct                   Ft <math>X^2</math>. Can be assertive.                   Ft <math>X^2</math></p>												
<p>(iii)</p>	<table border="0"> <tr> <td></td> <td>G<sub>1</sub></td> <td>G<sub>2</sub></td> <td>G<sub>3</sub></td> </tr> <tr> <td>O</td> <td>29</td> <td>37</td> <td>54</td> </tr> <tr> <td>E</td> <td>40</td> <td>40</td> <td>40</td> </tr> </table> <p><math>121/40 + 9/40 + 196/40</math>  <math>= 8.15</math>                  Using df = 2                  2.5% tables, 1.7% calculator</p>		G <sub>1</sub>	G <sub>2</sub>	G <sub>3</sub>	O	29	37	54	E	40	40	40	<p>M1                  A1                  M1                  A1                  M1                  A1 <b>6 (15)</b></p>	<p>For combining</p>
	G <sub>1</sub>	G <sub>2</sub>	G <sub>3</sub>												
O	29	37	54												
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<p>7(i)</p>	$F(t) = \begin{cases} 0 & t \leq 0, \\ t^4 & 0 < t \leq 1, \\ 1 & \text{otherwise.} \end{cases}$ <hr/>	<p>B1 B1 2</p> <hr/>	<p>For <math>t^4</math> For rest</p> <hr/>
<p>(ii)</p>	$\begin{aligned} G(h) &= P(H \leq h) \\ &= P(T \geq 1/h^{1/4}) \\ &= 1 - F(1/h^{1/4}) \\ &= 1 - 1/h \\ g(h) &= G'(h) \\ &= 1/h^2 \\ h &\geq 1, (0 \text{ otherwise}) \end{aligned}$ <hr/>	<p>M1 A1 A1 A1 M1 A1 B1 7</p> <hr/>	<p>Accept &lt;  With attempt at differentiation Only from G obtained correctly</p> <hr/>
<p>(iii)</p>	<p>EITHER: <math>\int_1^\infty (h^{-2} + 2h^{-3})dh</math>  <math>= [-h^{-1} - h^{-2}]_1^\infty</math>  <math>= 2</math>                  OR: <math>= 1 + 2 \int_1^\infty \frac{1}{h^3} dh</math>  <math>= 1 + 2 \left[ -\frac{1}{2h^2} \right]_1^\infty</math>  <math>= 2</math>                  OR: <math>E(1+2T^4) = 1 + \int_0^1 8t^7 dt</math>  <math>= 1 + [t^8]</math>  <math>= 2</math></p>	<p>M1 B1 A1 M1 B1 A1 M1 B1 A1 3 (12)</p> <hr/>	<p>For integrating <math>(1+2h^{-1})g(x)</math>, with limits from (ii)  Limits not required  Limits not required  Limits not required</p>