# Further Pure Mathematics FP3 (6669)

## Mock paper mark scheme

| Question number | Scheme | Marks |
|---|---|---|
| **1.** | $4\left(1-\operatorname{sech}^2 x\right)-2\operatorname{sech}^2 x=3$ | M1 |
| | $6\operatorname{sech}^2 x=1$ | A1 |
| | $\cosh x=\sqrt{6}$ | M1 A1 |
| | Using $\operatorname{arcosh} x=\ln\left(x+\sqrt{\left(x^2-1\right)}\right)$ | M1 |
| | $x=\pm\ln\left(\sqrt{6}+\sqrt{5}\right)$ | A1 (6) |
| | | **(6 marks)** |
| **2.** | $\dfrac{dy}{dx}=\sinh\left(\dfrac{1}{2}x\right)$ | B1 |
| | $\displaystyle\int y\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}dx = \int 2\cosh\left(\frac{1}{2}x\right)\sqrt{\left(1+\sinh^2\left(\frac{1}{2}x\right)\right)}dx$ | M1 A1 |
| | $\displaystyle= \int 2\cosh\left(\frac{1}{2}x\right)dx$ | |
| | $\displaystyle=\int\left(1+\cosh x\right)dx$ | |
| | $=x+\sinh x$ | M1 A1 |
| | $\displaystyle 2\pi\left[x+\frac{e^x-e^{-x}}{2}\right]_{-\ln 2}^{\ln 2} = \pi\left[\left(2\ln 2+2-\frac{1}{2}\right)-\left(-2\ln 2+\frac{1}{2}-2\right)\right]$ | M1 |
| | $=\pi\left(4\ln 2+3\right)$ | A1 (7) |
| | | **(7 marks)** |

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|---|---|---|
| **3.** | $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=-\dfrac{3\cosh\theta}{\sinh^2\theta}$ | B1 |
| | $\displaystyle\int\frac{1}{x\sqrt{(x^2+9)}}\,\mathrm{d}x=\int\frac{1}{\frac{3}{\sinh\theta}\sqrt{\left(\frac{9}{\sinh^2\theta}+9\right)}}\times\frac{-3\cosh\theta}{\sinh^2\theta}\,\mathrm{d}\theta$ | M1 A1 |
| | $=-\dfrac{1}{3}\displaystyle\int 1\,\mathrm{d}\theta=-\dfrac{1}{3}\theta$ | A1 |
| | $x=3\sqrt{3}\Rightarrow\sinh\theta=\dfrac{1}{\sqrt{3}}\Rightarrow\theta=\ln\left(\dfrac{1}{\sqrt{3}}+\dfrac{2}{\sqrt{3}}\right)=\ln\sqrt{3}$ | |
| | $x=4\Rightarrow\sinh\theta=\dfrac{3}{4}\Rightarrow\theta=\ln\left(\dfrac{3}{4}+\dfrac{5}{4}\right)=\ln 2$ | M1 A1 |
| | $\left[-\dfrac{1}{3}\theta\right]_{\ln 2}^{\ln\sqrt{3}}=\dfrac{1}{3}\left(\ln 2-\ln\sqrt{3}\right)=\dfrac{1}{3}\left(\dfrac{1}{2}\ln 4-\dfrac{1}{2}\ln 3\right)=\dfrac{1}{6}\ln\dfrac{4}{3}$ | M1 A1 (8) |
| | | **(8 marks)** |
| **4.** (a) | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{1+x}\times\dfrac{1}{2}x^{-\frac{1}{2}}\left(=\dfrac{1}{2x^{\frac{1}{2}}(1+x)}\right)$ | M1 A1 |
| | $x=\dfrac{1}{4}\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{4}{5}$ | A1 (3) |
| (b) | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2}(1+x)^{-1}x^{-\frac{1}{2}}$ | |
| | $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\dfrac{1}{2}(1+x)^{-2}\times x^{-\frac{1}{2}}-\dfrac{1}{4}(1+x)^{-1}\times x^{-\frac{3}{2}}$ | M1 A1 |
| | $=-\dfrac{1+3x}{4x^{\frac{3}{2}}(1+x)^2}$ | |
| | $2x(1+x)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+(1+3x)\dfrac{\mathrm{d}y}{\mathrm{d}x}$ | |
| | $=2x(1+x)\left(-\dfrac{1+3x}{4x^{\frac{3}{2}}(1+x)^2}\right)+(1+3x)\left(\dfrac{1}{2x^{\frac{1}{2}}(1+x)}\right)$ | M1 A1, A1 |
| | $=0$ ❋ | A1 cso (6) |
| | | **(9 marks)** |

| Question number | Scheme | Marks |
|---|---|---|
| **5.** (a) | $I_n = \int_0^{\frac{\pi}{2}} \sin x . \sin^{n-1} x \, \mathrm{d}x$ | |
| | $= \left[-\cos x \sin^{n-1} x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x . (n-1)\sin^{n-2} x \cos x \, \mathrm{d}x$ | M1 A1 |
| | $= 0 + \ldots$ | A1 |
| | $= (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x \left(1 - \sin^2 x\right) \mathrm{d}x$ | |
| | $= (n-1) I_{n-2} - (n-1) I_n$ | M1 |
| | Leading to $I_n = \dfrac{n-1}{n} I_{n-2}$ (✱) | A1 (5) |
| (b) | $\int_0^{\frac{\pi}{2}} x\left(\sin^5 x \cos x\right) \mathrm{d}x = \left[\dfrac{x \sin^6 x}{6}\right]_0^{\frac{\pi}{2}} - \dfrac{1}{6}\int_0^{\frac{\pi}{2}} \sin^6 x \, \mathrm{d}x$ | M1 A1 |
| | $I_6 = \dfrac{5}{6} \times \dfrac{3}{4} \times \dfrac{1}{2} I_0 = \dfrac{5}{6} \times \dfrac{3}{4} \times \dfrac{1}{2} \times \dfrac{\pi}{2} \left(= \dfrac{5\pi}{32}\right)$ | M1 A1 |
| | Hence $\int_0^{\frac{\pi}{2}} x\left(\sin^5 x \cos x\right) \mathrm{d}x = \dfrac{\pi}{12} - \dfrac{1}{6} \times \dfrac{5\pi}{32} = \dfrac{11\pi}{192}$ | A1 (5) |
| | | **(10 marks)** |

| Question number | | Scheme | Marks |
|---|---|---|---|
| **6.** | (a) | $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix}$ , $\overrightarrow{QR} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ | B1 |
| | | $\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & -4 \\ 2 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$ | M1<br>A2, 1, 0 (4) |
| | (b) | $\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = -2 \quad \Rightarrow \quad y + z = -2$ or equivalent | M1 A1 (2) |
| | (c) | $y + z = -2$ | |
| | | $x + y - z = 6$ | |
| | | Let $z = \lambda \quad \Rightarrow y = -\lambda - 2,\ x = 2\lambda + 8$ | M1 A1 A1 |
| | | $\mathbf{r} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ | M1 |
| | | $\left( \mathbf{r} - \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \right) \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \mathbf{0}$ | A1 (5) |
| | | | **(11 marks)** |

| Question number | Scheme | Marks |
|---|---|---|
| **7.** (a) | $\begin{pmatrix} 2 & k & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 3k+18 \\ 12 \\ -8 \end{pmatrix} = \lambda\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$ | M1 |
| | $\lambda = 4 \quad \Rightarrow \quad 3k+18=36 \quad \Rightarrow \quad k=6$ | M1 A1 (3) |
| (b) | $\lambda = 4$ is an eigenvalue | B1 |
| | $\begin{vmatrix} 2-\lambda & 6 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{vmatrix} = (1-\lambda)\left((2-\lambda)(1-\lambda)-6\right)$ | M1 |
| | $(\lambda-1)\left(\lambda^2-3\lambda-4\right) = (\lambda-1)(\lambda-4)(\lambda+1)$ | M1 |
| | $\lambda = (4,)\ 1,\ -1$      $1$ and $-1$ | A1 (4) |
| (c) | $\begin{pmatrix} 2 & 6 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}\begin{pmatrix} t-2 \\ t \\ 2t \end{pmatrix} = \begin{pmatrix} 8t-4 \\ 2t-2 \\ 0 \end{pmatrix}$ | M1 A2,1,0 |
| | $x = 8t-4,\ y = 2t-2,\ z = 0$ | |
| | $x - 4y - 4 = 0$ | M1 A1 (5) |
| | | **(12 marks)** |

| Question number | Scheme | Marks |
|---|---|---|
| **8.** (a) | $\frac{\mathrm{d}x}{\mathrm{d}u} = 5\sec u \tan u, \ \frac{\mathrm{d}y}{\mathrm{d}u} = 3\sec^2 u$ | |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\sec^2 u}{5\sec u \tan u} \quad \left(= \frac{3}{5\sin u}\right)$ | B1 |
| | $y - 3\tan u = \frac{3}{5\sin u}(x - 5\sec u)$ | M1 |
| | Leading to $3x = 5y\sin u + 15(\sec u - \tan u \sin u)$ | |
| | $3x = 5y\sin u + 15\left(\frac{1-\sin^2 u}{\cos u}\right)$ | |
| | $3x = 5y\sin u + 15\cos u \quad$ (✱) | M1 |
| | | A1 cso (4) |
| (b) | Equations of asymptotes $\quad y = \pm\frac{3}{5}x \quad$ both | B1 |
| | Eliminating $y$ or $x$ between $3x = 5y\sin u + 15\cos u$ and $y = \frac{3}{5}x$ | |
| | $3x = 3x\sin u + 15\cos u$ | |
| | $x = \frac{5\cos u}{1-\sin u}, \quad y = \frac{3\cos u}{1-\sin u}$ | M1 A1 |
| | Similarly between $3x = 5y\sin u + 15\cos u$ and $y = -\frac{3}{5}x$ | |
| | $x = \frac{5\cos u}{1+\sin u}, \quad y = -\frac{3\cos u}{1+\sin u}$ | M1 A1 |
| | Let $(x_M, y_M)$ be the coordinates of the mid-point of $RS$. | |
| | $x_M = \frac{1}{2}\left(\frac{5\cos u}{1-\sin u} + \frac{5\cos u}{1+\sin u}\right) = \frac{5\cos u}{2}\left(\frac{2}{1-\sin^2 u}\right) = 5\sec u$ | M1 |
| | $y_M = \frac{1}{2}\left(\frac{3\cos u}{1-\sin u} - \frac{3\cos u}{1+\sin u}\right) = \frac{3\cos u}{2}\left(\frac{2\sin u}{1-\sin^2 u}\right) = 3\tan u$ | A1 |
| | The coordinates $(x_M, y_M)$ are the same as $P$. | |
| | $P$ is the mid-point of $RS$. (✱) | A1 cso (8) |
| | | **(12 marks)** |