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**General Certificate of Education**

**Mathematics 6360**

**MPC4      Pure Core 4**

**Mark Scheme**

*2010 examination - January series*

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### Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

Q	Solution	Marks	Total	Comments
1(a)(i) (ii)  (b)	$f(-1) = -15 + 19 - 4 = 0$	B1	1	evaluate <b>or</b> complete division leading to a numerical remainder  Or decimal equivalent $(0.96 + 3.04 - 4)$ <b>or</b> zero remainder $\Rightarrow$ factor  Stated or implied.  Any appropriate method to find third factor  $\left. \begin{array}{l} (5x-2)(3x^2 \pm 5x \pm 2) + \text{attempt} \\ \text{to factorise} \\ \text{Factorise numerator correctly} \\ \text{and attempt to simplify} \end{array} \right\}$  CSO no ISW
	$f\left(\frac{2}{5}\right)$	M1		
	$\left(15 \times \frac{8}{125} + 19 \times \frac{4}{25} - 4\right) = 0 \Rightarrow$ factor	A1	2	
	$(x+1)$ is a factor	B1		
	Third factor is $(3x+2)$	M1 A1		
	$\frac{15x^2 - 6x}{f(x)} = \frac{3x(5x-2)}{(x+1)(5x-2)(3x+2)}$	M1		
	$= \frac{3x}{(x+1)(3x+2)}$	A1	5	
	<b>Total</b>		<b>8</b>	
2(a)  (b)(i)  (ii)  (c)	$R = \sqrt{10}$ $\tan \alpha = 3$ $\alpha = 1.249$ ignore extra out of range	B1 M1 A1	3	Accept $R = 3.16$ or better OE AWRT 1.25 SC $\alpha = 0.322$ B1 radians only  F on $R$  AWRT 4.39 $51.56^\circ$ or .. $.57^\circ$ or better  Two values, accept 2dp and condone 5.4 condone use of degrees  F on $x - \alpha$ , either value. AWRT  CSO 3dp or better
	minimum value $= -\sqrt{10}$	B1F	1	
	$\cos(x - \alpha) = -1$ $x = 4.391$	M1 A1F	2	
	$\cos(x - \alpha) = \frac{2}{\sqrt{10}}$ $x - \alpha = \pm 0.886$ 5.397 ignore extra out of range	M1 A1		
	$x = 0.36296..$ 2.13512.. $x = 0.363$ 2.135	A1F A1	4	
	<b>Total</b>		<b>10</b>	
(c)	<b>Alternative</b> $10 \sin^2 x - 12 \sin x + 3 = 0$  $\sin x =$ two numerical answers $-1 \leq \text{ans} \leq 1$  $x =$ one correct answer  $x = 0.363$ 2.135	M1 A1F A1F A1		Or equivalent quadratic using $\cos x$ (ie $\sin^2 x + \cos^2 x = 1$ used) Or equivalent using $\cos x$  CSO 3 dp or better

**MPC4 (cont)**

Q	Solution	Marks	Total	Comments			
<b>3(a)(i)</b>	$(1+x)^{\frac{1}{3}} = 1 \pm \frac{1}{3}x + kx^2$	M1	2	$1 \pm \frac{1}{3}x + kx^2$			
	$= 1 - \frac{1}{3}x + \frac{2}{9}x^2$	A1					
	<b>(ii)</b>	$\left(1 + \frac{3}{4}x\right)^{\frac{1}{3}} = 1 - \frac{1}{3} \times \frac{3}{4}x + \frac{2}{9}\left(\frac{3}{4}x\right)^2$	M1	2	$x$ replaced by $\frac{3}{4}x$ or start binomial again; condone missing brackets		
		$= 1 - \frac{1}{4}x + \frac{1}{8}x^2$	A1				
	<b>(b)</b>	$\sqrt[3]{\frac{256}{4+3x}} = k \left(1 + \frac{3}{4}x\right)^{\frac{1}{3}}$	M1	3	$k \neq 1$ F on (a)(ii) $k = 4$ , accept $\sqrt[3]{64}$ or $64^{\frac{1}{3}}$ CSO fully simplified Be convinced		
		$= 4 \left(1 - \frac{1}{4}x + \frac{1}{8}x^2\right)$	A1F				
$= 4 - x + \frac{1}{2}x^2$ or		A1					
$a = 4 \quad b = -1 \quad c = \frac{1}{2}$							
<b>Total</b>			<b>7</b>				
<b>4(a)</b>	$10x^2 + 8 = 2(x+1)(5x-1) +$ $A(5x-1) + B(x+1)$	M1	4	A and B terms correct  Use two values of $x$ to find A and B, or set up and solve $8 + 5A + B = 0$ $-2 - A + B = 8$ SC1 NWS A & B correct $\frac{4}{4}$ SC2 NWS A or B correct $\frac{1}{4}$			
	$x = -1 \quad x = \frac{1}{5}$	A1					
	$A = -3 \quad B = 7$	m1					
		A1					
	<b>(b)</b>	$\int \frac{10x^2 + 8}{(x+1)(5x-1)} dx = \int 2 - \frac{3}{x+1} + \frac{7}{5x-1} dx$			M1	4	Use the partial fractions  $a \ln(x+1) + b \ln(5x-1)$ condone missing brackets  F on A and B
		$= 2x + C$			B1		
$-3 \ln(x+1) + \frac{7}{5} \ln(5x-1)$		M1					
<b>Total</b>			<b>8</b>				
<b>5</b>	$x^2 + xy = e^y$	B1	5	$2x$ Use product rule  RHS  CSO			
	$2x + y + x \frac{dy}{dx} = e^y \frac{dy}{dx}$	M1					
		A1					
		B1					
	$(-1, 0) \quad \frac{dy}{dx} = -1$	A1					
<b>Total</b>			<b>5</b>				

**MPC4 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>6(a)(i)</b>	$\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$	B1 B1	2	OE condone use of $x$ etc, but variable must be consistent
<b>(ii)</b>	$\sin \theta = \frac{4}{5} \Rightarrow \sin 2\theta = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$ <p>or</p> $2 \times \sin \left( \cos^{-1} \frac{3}{5} \right) \times \frac{3}{5}$ $\cos 2\theta = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$	B1		AG Use of $106.26^\circ \dots$ B0
<b>(b)(i)</b>	$\frac{dx}{d\theta} = 6 \cos 2\theta \quad , \quad \frac{dy}{d\theta} = -8 \sin 2\theta$ $\frac{dy}{dx} = -\frac{4 \sin 2\theta}{3 \cos 2\theta} \quad \text{ISW}$	M1 A1  A1	3	Attempt both derivatives. ie $p \cos 2\theta$ Both correct. $q \sin 2\theta$ CSO OE
<b>(ii)</b>	$P \left( \frac{72}{25}, -\frac{28}{25} \right)$ $\text{Gradient} = -\frac{4}{3} \times -\frac{24}{7}$	B1F  M1		(2.88, -1.12) Their $\frac{q \sin 2\theta}{p \cos 2\theta}$ or $\frac{p \cos 2\theta}{q \sin 2\theta}$ must be working with rational numbers
	$\text{Tangent } y + \frac{28}{25} = \frac{32}{7} \left( x - \frac{72}{25} \right) \quad \text{ISW}$	A1	3	Any correct form. $7y = 32x - 100$ Fractions in simplest form Equation required
<b>Total</b>			<b>10</b>	

**MPC4 (cont)**

Q	Solution	Marks	Total	Comments
7	$\int y \, dy = \int \cos\left(\frac{x}{3}\right) dx$ $\frac{1}{2}y^2 = 3\sin\left(\frac{x}{3}\right) + C$ $\left(\frac{\pi}{2}, 1\right) \quad \frac{1}{2} = 3\sin\frac{\pi}{6} + C$ $C = -1$ $y^2 = 6\sin\left(\frac{x}{3}\right) - 2$	<p>B1</p> <p>B1 B1</p> <p>M1</p> <p>A1F</p> <p>A1</p>	6	<p>Separate; condone missing integral signs.</p> <p>Accept <math>\frac{\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}</math></p> <p>Use <math>\left(\frac{\pi}{2}, 1\right)</math> to find C</p> <p>must be in form <math>py^2 = q\sin\left(\frac{x}{3}\right) + C</math></p> <p>CSO</p>
<b>Total</b>			<b>6</b>	
8(a)	$0 = 2 + \lambda \Rightarrow \lambda = -2$ <b>Check</b> $-1 + -2 \times -3 = -1 + 6 = 5$ $-5 - 2 \times 2 = -5 \times -4 = -9$	M1  A1	2	OE
(b)	$\overrightarrow{BC} = \begin{bmatrix} 9 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 5 \\ -9 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \\ 12 \end{bmatrix}$	M1 A1	2	$\pm (\overrightarrow{OC} - \overrightarrow{OB})$
(c)(i)	$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + 2\overrightarrow{BC}$ $\overrightarrow{OD} = \begin{bmatrix} 2 \\ -1 \\ -5 \end{bmatrix} + 2\begin{bmatrix} 9 \\ -3 \\ 12 \end{bmatrix} = \begin{bmatrix} 20 \\ -7 \\ 19 \end{bmatrix}$ <p>D is (20, -7, 19)</p>	M1  A1	2	AG
(ii)	$\overrightarrow{PD} = \overrightarrow{OD} - \overrightarrow{OP} =$ $\begin{bmatrix} 20 \\ -7 \\ 19 \end{bmatrix} - \begin{bmatrix} 2+p \\ -1-3p \\ -5+2p \end{bmatrix} = \begin{bmatrix} 18-p \\ -6+3p \\ 24-2p \end{bmatrix}$ $\overrightarrow{PD} \cdot \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = 0$ $(18-p) \times 1 + (-6+3p) \times -3 + (24-2p) \times 2 = 0$ $p = 6$	M1  A1  B1  m1 A1	5	<p>Find <math>\overrightarrow{PD}</math> in terms of p</p> <p>condone <math>\overrightarrow{PD} = \overrightarrow{OP} - \overrightarrow{OD}</math> here</p> <p>CSO OE working with <math>\overrightarrow{DP}</math></p>
<b>Total</b>			<b>11</b>	

**MPC4 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>9(a)(i)</b>	$t = 0 \quad h = A(1 - 1) = 0$	B1	1	
<b>(ii)</b>	$57 = A \left( 1 - e^{-\frac{12}{4}} \right)$ $A = \frac{57}{(1 - e^{-3})} \approx 60$	M1 A1	2	Or 59.9... seen. A = correct expression $\approx 60$ 2sf
<b>(b)(i)</b>	$h = 48 \quad \frac{48}{60} = 1 - e^{-\frac{1}{4}t}$ $\ln \left( e^{-\frac{1}{4}t} \right) = \ln \left( \frac{1}{5} \right)$ $-\frac{1}{4}t = -\ln 5 \Rightarrow t = 4 \ln 5$	M1 m1 A1	3	
<b>(ii)</b>	$\frac{dh}{dt} = -\frac{1}{4} \times -60 \times e^{-\frac{1}{4}t}$ $60e^{-\frac{1}{4}t} = 60 - h \Rightarrow \frac{dh}{dt} = \frac{1}{4}(60 - h)$ $\frac{dh}{dt} = 15 - \frac{h}{4}$	M1 m1 A1	3	Differentiate, condone sign errors Eliminate $e^{-\frac{1}{4}t}$ CSO, AG
<b>(iii)</b>	$h = 8$	B1	1	
	<b>Total</b>		<b>10</b>	
	<b>TOTAL</b>		<b>75</b>	