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Mark Scheme (Results)

Summer 2009

GCE

GCE Mathematics (6690/01)

A PEARSON COMPANY

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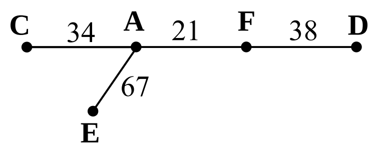
6690 Decision Mathematics D2
Mark Scheme

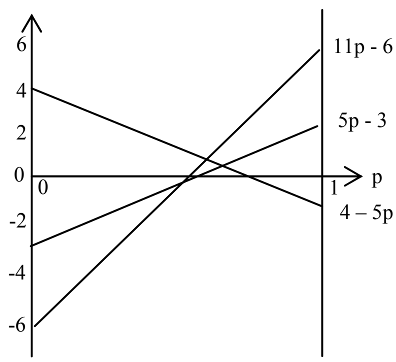
Question Number	Scheme	Marks
Q1 (a)	There are more tasks than people.	B1 (1)
(b)	Adds a row of zeros	B1 (1)
(c)	$\begin{bmatrix}15&11&14&12\\13&8&17&13\\14&9&13&15\\0&0&0&0\end{bmatrix}\rightarrow\begin{bmatrix}4&0&3&1\\5&0&9&5\\5&0&4&6\\0&0&0&0\end{bmatrix};\rightarrow\begin{bmatrix}3&0&2&0\\4&0&8&4\\4&0&3&5\\0&1&0&0\end{bmatrix}$	B1;M1A1
	Either $\begin{bmatrix}3&3&2&0\\1&0&5&1\\1&0&0&2\\0&4&0&0\end{bmatrix}$	M1 A1
	Or $\begin{bmatrix}1&0&0&0\\2&0&6&4\\2&0&1&5\\0&3&0&2\end{bmatrix}\rightarrow\begin{bmatrix}1&1&0&0\\1&0&5&3\\1&0&0&4\\0&4&0&2\end{bmatrix}$	
	J – 4, M – 2, R – 3, (D – 1)	A1 (6)
(d)	Minimum cost is (£)33.	B1 (1)
		[9]

Question Number	Scheme	Marks
Q2 (a)	In the classical problem each vertex must be visited **only** once. In the practical problem each vertex must be visited **at least** once.	B2, 1, 0 (2)
(b)	A F D B E C A {1 4 6 3 5 2 } 21 + 38 + 58 + 36 + 70 + 34 = 257	M1 A1 A1 (3)
(c)	257 is the better upper bound, it is lower.	B1ft (1)
(d)	R.M.S.T.  Lower bound is 160 + 36 + 58 = 254	M1 A1 M1A1 (4)
(e)	Better lower bound is 254, it is higher	B1ft
(f)	$254 < \text{optimal} \le 257$	B1 (2)
		[12]
	Notes: (a) 1B1: Generous, on the right lines bod gets B1 2B1: cao, clear answer. (b) 1M1:Nearest Neighbour each vertex visited once (condone lack of return to start) 1A1: Correct route cao – must return to start. 2A1: 257 cao (c) 1B1ft: ft their lowest. (d) 1M1: Finding correct RMST (maybe implicit) 160 sufficient 1A1: cao tree or 160. 2M1:Adding 2 least arcs to B, 36 and 58 only 2A1: 254 (e) 1B1ft: ft their highest (f) 1B1: cao	

Question Number	Scheme	Marks
Q3 (a)	Row minima {-5, -4, -2} row maximin = -2 Column maxima {1, 6, 13} col minimax = 1 $-2 \neq 1$ therefore not stable.	M1 A1 A1 (3)
(b)	Column 1 dominates column 3, so column 3 can be deleted.	B1 (1)
(c)	see table below	B1 B1 (2)

	A plays 1	A plays 2	A plays 3
B plays 1	5	-1	2
B plays 2	-6	4	-3

Question Number	Scheme	Marks
(d)	Let B play row 1 with probability p and row 2 with probability (1-p) If A plays 1, B's expected winnings are $11p - 6$ If A plays 2, B's expected winnings are $4 - 5p$ If A plays 3, B's expected winnings are $5p - 3$	M1 A1
	graph below	M1 A1



Question Number	Scheme	Marks
	$5p - 3 = 4 - 5p$ $10p = 7$ $p = \frac{7}{10}$	M1
	B should play 1 with a probability of 0.7 2 with a probability of 0.3 and never play 3	A1
	The value of the game is 0.5 to B	A1 (7)
		[13]

Question Number	Scheme	Marks
Q4 (a)	Value of cut $C_1 = 34$; Value of cut $C_2 = 45$	B1; B1 (2)
(b)	S B F G T or S B F E T – value 2 Maximum flow = 28	M1 A1 A1=B1 (3) **[5]**
	Notes: (a) 1B1: cao 2B1: cao (b) 1M1: feasible flow-augmenting route and a value stated 1A1: a correct flow-augmenting route and value 1A1= B1: cao	
Q5 (a)	$x = 0,\ y = 0,\ z = 2$	B2,1,0 (2)
(b)	$P - 2x - 4y + \frac{5}{4}r = 10$	M1 A1 (2) **[4]**
	Notes: (a) 1B1: Any 2 out of 3 values correct 2B1: All 3 values correct. (b) 1M1: One equal sign, modulus of coefficients correct. All the right ingredients. 1A1: cao – condone terms of zero coefficient	

Question Number	Scheme	Marks
Q6 (a)	The supply is equal to the demand	B1 (1)
(b)	see table below	B1 (1)

	A	B	C
X	16	6	
Y		9	8
Z			15

(c)

	A	B	C
X	16- θ	6+ θ	
Y		9- θ	8+ θ
Z	θ		15- θ

M1 A1

Value of θ = 9, exiting cell is YB — A1 (3)

(d)

		17	8	20
		A	B	C
0	X	7	15	
-5	Y			17
-11	Z	9		6

M1 A1

XC = 7 – 0 – 20 = -13
YA = 16 + 5 – 17 = 4
YB = 12 + 5 – 8 = 9
ZB = 10 + 11 – 8 = 13

A1 (3)

	A	B	C
X	7- θ	15	θ
Y			17
Z	9+ θ		6- θ

Value of θ = 6, entering cell XC, exiting cell ZC — M1 A1

	A	B	C
X	1	15	6
Y			17
Z	15		

A1 (3)

Cost (£) 524 — B1 (1)

[12]

Question Number	Scheme	Marks

Q7

(a)

Stage	State (in £1000s)	Action (in £1000s)	Dest. (in £1000s)	Value (in £1000s)	Marks
	250	250	0	300*	
1	200	200	0	240*	
	150	150	0	180*	
	100	100	0	120*	
	50	50	0	60*	
	0	0	0	0*	
	250	280	0	200 + 0 = 280	
		200	50	235 + 60 = 295	
		150	100	190 + 120 = 310*	
		100	150	125 + 180 = 305	1M1 A1
		50	200	65 + 240 = 305	
		0	250	0 + 300 = 300	
2	200	200	0	235 + 0 = 235	
		150	50	190 + 60 = 250*	
		100	100	125 + 120 = 245	A1
		50	150	65 + 180 = 245	
		0	200	0 + 240 = 240	
	150	150	0	190 + 0 = 190*	2M1
		100	50	125 + 60 = 185	
		50	100	65 + 120 = 185	A1
		0	150	0 + 180 = 180	
	100	100	0	125 + 0 = 125*	A1
		50	50	65 + 60 = 125*	
		0	100	0 + 120 = 120	
	50	50	0	65 + 0 = 65*	
		0	50	0 + 60 = 60	
	0	0	0	0 + 0 = 0*	3M1
3	250	250	0	300 + 0 = 300	A1ft
		200	50	230 + 65 = 295	
		150	100	170 + 125 = 295	
		100	150	110 + 190 = 300	
		50	200	55 + 250 = 305	
		0	250	0 + 310 = 310*	

Maximum income £310 000 — B1

Scheme	1	2	3
Invest (in £1000s)	100	150	0

B1 (10)

(b)

Stage: Scheme being considered — B1
State: Money available to invest — B1
Action: Amount chosen to invest — B1 (3)

[13]

Question Number	Scheme	Marks
Q8	E.g. Add 6 to make all elements positive $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$	B1
	Let Laura play 1, 2 and 3 with probabilities p_1, p_2 and p_3 respectively Let V = value of game + 6	B1
	e.g. Maximise P = V Subject to:	B1
	$V - 4p_1 - 13p_2 - 7p_3 \le 0$ $V - 14p_1 - 10p_2 - p_3 \le 0$ $V - 5p_1 - 3p_2 - 10p_3 \le 0$ $p_1 + p_2 + p_3 \le 1$ $p_1, p_2, p_3 \ge 0$	M1 A3,2ft,1ft,0 (7)
	Notes: 1B1: Making all elements positive 2B1: Defining variables 3B1: Objective, cao word and function 1M1: At least one constraint in terms of their variables, must be going down columns. Accept = here. 1A1ft: ft their table. One constraint in V correct. 2A1ft: ft their table. Two constraints in V correct. 3A1: CAO all correct .	**[7]**
	Alt using x_i method Now additionally need: let $x_i = \frac{p_i}{v}$ for 2B1 minimise $(P) = x_1 + x_2 + x_3 = \frac{1}{v}$ subject to: $4x_1 + 13x_2 + 7x_3 \ge 1$ $14x_1 + 10x_2 + x_3 \ge 1$ $5x_1 + 3x_2 + 10x_3 \ge 1$ $x_i \ge 0$	