



General Certificate of Education
Advanced Level Examination
January 2012

Mathematics

MFP3

Unit Further Pure 3

Monday 23 January 2012 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = \frac{y - x}{y^2 + x}$$

and

$$y(1) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$.

(3 marks)

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to $y(1.2)$, giving your answer to three decimal places.

(3 marks)

2 Find

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{4+x} - 2}{x + x^2} \right] \quad (3 \text{ marks})$$

3 Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 26e^x$$

given that $y = 5$ and $\frac{dy}{dx} = 11$ when $x = 0$. Give your answer in the form

$y = f(x)$.

(10 marks)



- 4 (a) By using an integrating factor, find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \ln x \quad (7 \text{ marks})$$

- (b) Hence, given that $y \rightarrow 0$ as $x \rightarrow 0$, find the value of y when $x = 1$. (3 marks)
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- 5 (a) Explain why $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2+3e^{4x}} dx$ is an improper integral. (1 mark)

- (b) By using the substitution $u = x^2e^{-4x} + 3$, find

$$\int \frac{x(1-2x)}{x^2+3e^{4x}} dx \quad (3 \text{ marks})$$

- (c) Hence evaluate $\int_{\frac{1}{2}}^{\infty} \frac{x(1-2x)}{x^2+3e^{4x}} dx$, showing the limiting process used. (4 marks)
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- 6 (a) Given that $y = \ln \cos 2x$, find $\frac{d^4y}{dx^4}$. (6 marks)

- (b) Use Maclaurin's theorem to show that the first two non-zero terms in the expansion, in ascending powers of x , of $\ln \cos 2x$ are $-2x^2 - \frac{4}{3}x^4$. (3 marks)

- (c) **Hence** find the first two non-zero terms in the expansion, in ascending powers of x , of $\ln \sec^2 2x$. (2 marks)



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- 7 It is given that, for $x \neq 0$, y satisfies the differential equation

$$x \frac{d^2y}{dx^2} + 2(3x + 1) \frac{dy}{dx} + 3y(3x + 2) = 18x$$

- (a) Show that the substitution $u = xy$ transforms this differential equation into

$$\frac{d^2u}{dx^2} + 6 \frac{du}{dx} + 9u = 18x \quad (4 \text{ marks})$$

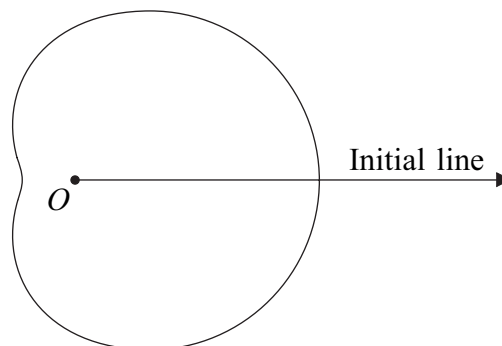
- (b) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} + 2(3x + 1) \frac{dy}{dx} + 3y(3x + 2) = 18x$$

giving your answer in the form $y = f(x)$. (8 marks)

- 8 The diagram shows a sketch of the curve C with polar equation

$$r = 3 + 2 \cos \theta, \quad 0 \leq \theta \leq 2\pi$$



- (a) Find the area of the region bounded by the curve C . (6 marks)
- (b) A circle, whose cartesian equation is $(x - 4)^2 + y^2 = 16$, intersects the curve C at the points A and B .
- (i) Find, in surd form, the length of AB . (6 marks)
- (ii) Find the perimeter of the segment AOB of the circle, where O is the pole. (3 marks)

