4733 Probability & Statistics 2

1	$\frac{105.0-\mu}{\sigma} = -0.7$; $\frac{110.0-\mu}{\sigma} = -0.5$ Solve: $\sigma = 25$ $\mu = 122.5$	M1 A1 B1 M1 A1 A1	 6	Standardise once, equate to Φ^{-1}, allow σ^2 Both correct including signs & σ, no cc (continuity correction), allow wrong z Both correct z-values. "1 –" errors: M1A0B1 Get either μ or σ by solving simultaneously σ a.r.t. 25.0 $\mu = 122.5 \pm 0.3$ or 123 if clearly correct, allow from σ^2 but *not* from $\sigma = -25$.
2	Po(20) ≈ N(20, 20) Normal approx. valid as $\lambda > 15$ $1-\Phi\left(\frac{24.5-20}{\sqrt{20}}\right) = 1 - \Phi(1.006)$ $= 1 - 0.8427$ **= 0.1573**	M1 A1 B1 M1 A1 A1	 6	Normal stated or implied (20, 20) or (20, √20) or (20, 20^2), can be implied "Valid as $\lambda > 15$", *or* "valid as λ large" Standardise 25, allow wrong or no cc, √20 errors $1.0 < z \le 1.01$ Final answer, art 0.157
3	$H_0: p = 0.6$, $H_1: p < 0.6$ where p is proportion in population who believe it's good value $R \sim B(12, 0.6)$ α: $P(R \le 4) = 0.0573$ > 0.05	B2 M1 A1 B1		Both, B2. Allow π, % One error, B1, except x or $\bar{x}$ or r or R: 0 B(12, 0.6) stated or implied, e.g. N(7.2, 2.88) *Not* P(< 4) or P(≥ 4) or P(= 4) Must be using P(≤ 4), or P(> 4) < 0.95 and binomial
	β: CR is ≤ 3 and 4 > 3 $p = 0.0153$	B1 A1		Must be using CR; explicit comparison needed
	Do not reject H_0. Insufficient evidence that the proportion who believe it's good value for money is less than 0.6	M1 A1	 7	Correct conclusion, needs B(12,0.6) and ≤ 4 Contextualised, some indication of uncertainty [**SR**: N(7.2, …) or Po(7.2): poss B2 M1A0] [**SR**: P(< 4) or P(= 4) or P(≥ 4): B2 M1A0]
4 (i)	Eg "not all are residents"; "only those in street asked"	B1 B1	 2	One valid relevant reason A definitely different valid relevant reason *Not* "not a random sample", *not* "takes too long"
(ii)	Obtain list of whole population Number it sequentially Select using random numbers [Ignore method of making contact]	B1 B1 B1	 3	"Everyone" or "all houses" must be implied *Not* "number it with random numbers" unless then "arrange in order of random numbers" **SR**: "Take a random sample": B1 **SR**: Systematic: B1 B0, B1 if start randomly chosen
(iii)	Two of: α: Members of population equally likely to be chosen β: Chosen independently/randomly γ: Large sample (e.g. > 30)	B1 B1	 2	One reason. NB : If "independent", must be "chosen" independently, not "views are independent" Another reason. Allow "fixed sample size" but not both that and "large sample". Allow "houses"

5	(i)	Bricks scattered at constant average rate & independently of one another	B1 B1 **2**	B1 for each of 2 different reasons, in context. (Treat "randomly" ≡ "singly" ≡ "independently")
	(ii)	Po(12) P(≤ 14) – P(≤ 7) [= .7720 – .0895] [or P(8) + P(9) + … + P(14)] **= 0.6825**	B1 M1 A1 **3**	Po(12) stated or implied Allow one out at either end or both, eg 0.617, or wrong column, but *not* from Po(3) nor, eg, .9105 – .7720 Answer in range [0.682, 0.683]
	(iii)	$e^{-\lambda} = 0.4$ $\lambda = -\ln(0.4)$ = 0.9163 Volume = 0.9163 ÷ 3 = **0.305**	B1 M1 A1 M1 **4**	This equation, aef, can be implied by, eg 0.9 Take ln, or 0.91 by T & I λ art 0.916 or 0.92, can be implied Divide their λ value by 3 [**SR**: Tables, eg 0.9÷3: B1 M0 A0 M1]
6	(i)	33.6 $\frac{115782.84}{100} - 33.6^2$ [= 28.8684] $\times \frac{100}{99}$ **= 29.16**	B1 M1 M1 A1 **4**	33.6 clearly stated [not recoverable later] Correct formula used for biased estimate $\times \frac{100}{99}$, M's independent. Eg $\frac{\Sigma r^2}{99}[-33.6^2]$ **SR** B1 variance in range [29.1, 29.2]
	(ii)	$\bar{R} \sim N(33.6, 29.16/9)$ $= N(33.6, 1.8^2)$ $1-\Phi\left(\frac{32-33.6}{\sqrt{3.24}}\right)$ [= Φ(0.8889)] **= 0.8130**	M1 A1 M1 A1 **4**	Normal, their μ, stated or implied Variance [their (i)]÷9 [*not* ÷100] Standardise & use Φ, 9 used, answer > 0.5, allow √ errors, allow cc 0.05 but *not* 0.5 Answer, art 0.813
	(iii)	No, distribution of R is normal so that of $\bar{R}$ is normal	B2 **2**	Must be saying this. Eg "9 is not large enough": B0. Both: B1 max, unless saying that n is irrelevant.
7	(i)	$\frac{2}{9}\int_0^3 x^3(3-x)dx = \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$ [= 2.7] – $(1½)^2$ $= \frac{9}{20}$ or **0.45**	M1 A1 B1 M1 A1 **5**	Integrate $x^2 f(x)$ from 0 to 3 [*not* for μ] Correct indefinite integral Mean is 1½, soi [not recoverable later] Subtract their μ^2 Answer art 0.450
	(ii)	$\frac{2}{9}\int_0^{0.5} x(3-x)dx = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^{0.5}$ $= \frac{2}{27}$ AG	M1 A1 **2**	Integrate f(x) between 0, 0.5, must be seen somewhere Correctly obtain given answer $\frac{2}{27}$, decimals other than 0.5 not allowed, 1 more line needed (eg [ ] = ⅓)
	(iii)	B(108, $\frac{2}{27}$) ≈ N(8, 7.4074) $1-\Phi\left(\frac{9.5-8}{\sqrt{7.4074}}\right)$ = 1 – Φ(0.5511) **= 0.291**	B1 M1 A1 M1 A1 A1 **6**	B(108, $\frac{2}{27}$) seen or implied, eg Po(8) Normal, mean 8 … … variance (or SD) 200/27 or art 7.41 Standardise 10, allow √ errors, wrong or no cc, needs to be using B(108,…) Correct √ and cc Final answer, art 0.291

	(iv)	$\overline{X} \sim N(1.5, \frac{1}{240})$	B1 B1√ B1√ **3**	Normal NB: *not* part (iii) Mean their μ Variance or SD (their 0.45)/108 [not (8, 50/729)]
8	(i)	$H_0 : \mu = 78.0$ $H_1 : \mu \neq 78.0$ $z = \frac{76.4 - 78.0}{\sqrt{68.9/120}} = -2.1115$ > -2.576 or $0.0173 > 0.005$	B1 B1 M1 A1 B1	Both correct, B2. One error, B1, but x or $\bar{x}$: B0. Needs ±(76.4 – 78)/√(σ÷120), allow √ errors art –2.11, or p = 0.0173 ± 0.0002 Compare z with (–)2.576, or p with 0.005
		$78 \pm z\sqrt{(68.9/120)}$ $= 76.048$ $76.4 > 76.048$	M1 A1√ B1	Needs 78 and 120, can be – only Correct CV to 3 sf, √ on z z = 2.576 and compare 76.4, allow from 78 ↔ 76.4
		Do not reject H_0. Insufficient evidence that the mean time has changed	M1 A1√ **7**	Correct comparison & conclusion, needs 120, "like with like", correct tail, $\bar{x}$ and μ right way round Contextualised, some indication of uncertainty
	(ii)	$\frac{1}{\sqrt{68.9/n}} > 2.576$ $\sqrt{n} > 21.38$, $n_{\min} = \mathbf{458}$ Variance is estimated	M1 M1 A1 B1 **4**	IGNORE INEQUALITIES THROUGHOUT Standardise 1 with n and 2.576, allow √ errors, cc etc but *not* 2.326 Correct method to solve for √n (*not* from n) 458 only (*not* 457), *or* 373 from 2.326, signs correct Equivalent statement, allow "should use t". In principle nothing superfluous, but "variance stays same" B1 bod