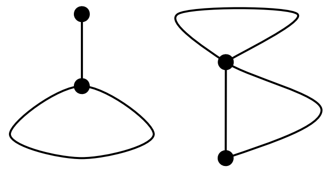
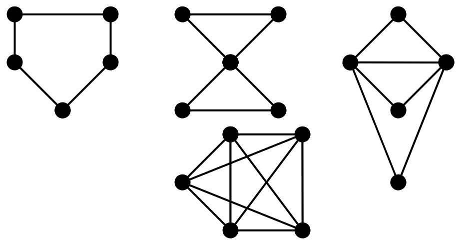
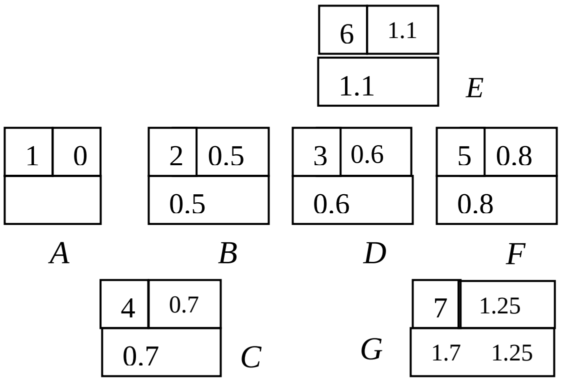
1(i) (a)	31 75 87 42 43 70 56 61 95 28 (may be shown vertically or as separate swaps) 9 comparisons and 8 swaps The smallest (final) mark, 28	M1 A1 B1 B1	**[4]**	28 moved to the end of the list, no other values moved Correct list at end of first pass (cao) 9 and 8 (written, not tallies) (cao) - if not specified, assume the larger value is comparisons (their) 28 or smallest/least or final/last/end If sorted into increasing order: 28 31 75 42 43 70 56 61 87 95 M0 A0, then 9 and 6 = B1 and (their) 95 or largest/greatest/biggest or final/last/end = B1
(b)	75 87 42 43 70 56 61 95 31 28	B1	**[1]**	Correct list at end of second pass If sorted into increasing order and already penalised in (i)(a) then condone here: 28 31 42 43 70 56 61 75 87 95
(c)	7 more passes	B1	**[1]**	7 (cao)
(ii)	31 28 75 87 42 43 70 56 61 95 75 31 28 87 42 43 70 56 61 95 1 comparison and 0 swaps in first pass 2 comparisons and 2 swaps in second pass	M1 A1 B1 B1	**[4]**	31 28 75 or 31 28 75 … Correct list, in full, at end of second pass Lists must be easily found, not picked out from working, if the candidate has labelled passes use them as labelled 1 and 0 (written)(cao) may appear next to list 2 and 2 (written)(cao) may appear next to list If sorted into increasing order: 28 31 75 … M0, A0, then 1 and 1 = B1; 1 and 0 = B1
(iii)	Bubble sort does not terminate early, since it takes 9 passes to get 95 to the front of the list, so it uses 9+8+…+1 or 45 comparisons Shuttle sort takes fewer than 1+2+…+9 comparisons, since, for example, in the fourth pass 42 will be compared with 28, 31 and 75 but not with 87.	B1 B1	**[2]**	Identifying that bubble sort <u>does not terminate early</u> (Just stating 9+8+…+1 or 45 = B0) Allow 'the largest number is at the end of the list' or '95 at end' A good explanation of why shuttle sort requires fewer comparisons <u>in this particular case</u> Do not accept 'because the list is not in reverse order'
(iv)	$20\times\left(\frac{50}{10}\right)^2$ = 500 seconds	M1 A1	**[2]**	Correct method 500 seconds or 8 mins 20 sec (without wrong working)

2(i)	Cannot have an odd number of odd nodes Odd vertices come in pairs	B1	**[1]**	Sum of orders must be even Sum of orders is 9 so 4.5 arcs (which is impossible)
(ii)	eg  Many other correct possibilities	M1 A1	 **[2]**	A diagram showing a graph with four vertices that is not connected and not simple Vertices have orders 1, 2, 3, 4
(iii)	The vertex of order 4 needs to connect to four other vertices, but there are only three other vertices available, so one vertex must be joined twice or the vertex of order 4 is connected to itself. Hence the graph cannot be simple	M1 A1	 **[2]**	Specifically identifying that the problem is with the vertex of order 4 Explaining why the graph cannot be simple (either reason) and stating that simple cannot be achieved Ignore any claims about whether or not the graph is connected
(iv) (a)	Each vertex of order 4 connects to each of the others, since graph is simple. Hence the other two vertices must have order (at least) 3. But Eulerian, so all must have order 4.	B1	**[1]**	Any reasonable explanation, but not just a diagram of a specific case 'the other two must be odd but they can't because Eulerian' is not enough Note: the graph has five vertices
(b)	Graph is Eulerian - so each vertex order is even; simple - so no vertex has order more than 4; and connected - so no vertex has order 0. Hence each vertex has order either 2 or 4. But cannot have 3 or 4 vertices of order 4. So must have 0, 1, 2 or 5 vertices of order 4. 	B1 M1 A1	 **[3]**	Explaining why there are only four such graphs Or list all the possibilities (eg 22222 42222 44222 44444) Any two correct (note: must be simply connected and Eulerian) All four correct and no extras (apart from topologically equivalent variations)

3(i)	$y \geq x$ $x \geq 0$ $y \leq 7 - \frac{2}{3}x$	M1 M1 A1	 **[3]**	Boundaries $y = x$ and $x = 0$ in any form (may be shown as an equality or an inequality with inequality sign wrong) Boundary $2x + 3y = 21$ in any form <u>All</u> inequalities correct (and any extras do not affect the feasible region)
(ii)	$(0, 7) \Rightarrow 42$ $(4.2, 4.2) \Rightarrow 29.4$ or $(\frac{21}{5}, \frac{21}{5}) \Rightarrow \frac{147}{5}$ At optimum, $x = 0$ and $y = 7$ $P_1 = 42$	M1 A1 A1	 **[3]**	Substantially correct attempt at testing vertices (at least one vertex apart from (0, 0)) or using a line of constant profit (may be implied) Accept (0, 7) identified (cao) 42 (stated) (cao) NOT deduced from earlier working, unless identified
(iii)	$(4.2, 4.2)$ $P_k = 4.2(k + 6)$ or $4.2k + 25.2$	B1 B1	 **[2]**	cao cao
(iv)	Compare $kx + 6y$ with boundary $2x + 3y$ or algebraically, $4.2(k + 6)$ with 42 or $-\frac{k}{6}$ with $-\frac{2}{3}$ $\Rightarrow k \leq 4$ $k \leq 4$ or $k < 4$ implies M1, A1	M1 A1	 **[2]**	Algebraically or using line, <u>or implied</u> (allow = here) Accept $k < 4$ No need to say that $k > 0$, but candidates may also say $k > 0$ or $k \geq 0$ Note: k is continuous, so answers such as 'k = 1, 2, 3, 4' or 'k = 1, 2, 3', with no other working, would get M1, A0

4(i)	 Route: $A – B – D – F$ - G	M1 A1 B1 B1 B1	[5]	1.7 shown as a temporary label at G All temporary labels correct with no extras (may not have written temporary label when it becomes permanent) All permanent labels correct (cao) Order of labelling correct (cao) This route written down (not reversed) (cao)
(ii)	Route Inspection problem	B1	**[1]**	Accept Chinese postman Allow ‘postman’, ‘postman route’, but not just ‘inspection’
(iii)	*CD* (*CBD*) = 0.3, *DG* (*DFG*) = 0.65, *CG* (*CBDFG*) = 0.95 *CD* (*CBD*) and *FG* = 0.75 or *CD* (*CBD*) and *EG* (*EFG*) = 1.05 Length = 3.7 + 0.5 + 0.3 + 0.75 = 5.25 km	M1 A1 M1 M1 A1	**[5]**	Any one of these seen (explicitly or as part of a calculation) All three of these seen (explicitly or as parts of calculations) Or either of these with *AB* to give 1.25 or 1.55 respectively Adding their 0.75 to 3.7 or their 0.75 to 3.7 + 0.5 + 0.3 (cao) units not needed 5.25 implies M1, M1 A1, irrespective of working
(iv)	*B* – *D* - *F* – *G* – *C* – *B* 1.9 km	B1 B1	**[2]**	cao 1.9 (cao) irrespective of method
(v)	[TREE] Vertices added in order *BDCF* or *BDFC* Arcs added in order *BD*, *BC*, *DF* or *BD*, *DF*, *BC* Two shortest arcs from *G* total 0.45 + 0.65 = 1.1 Lower bound = 0.5 + 1.1 = 1.6 km	B1 B1 M1 A1	**[4]**	Correct tree drawn A valid order of adding vertices or a valid order of adding arcs 0.45 and 0.65, or total 1.1 (may be implied from 1.6) 1.6 (cao) units not needed 1.6 implies M1, A1

5(i)

Answer	Marks	Total	Guidance
$600x + 800y + 500z \le 5000$	M1		Correct inequality, allow < for M mark only
$\Rightarrow 6x + 8y + 5z \le 50$	A1		Correct fully simplified form (cao)
$120x + 80y + 120z \le 800$	M1		Correct inequality, allow < for M mark only
$\Rightarrow 3x + 2y + 3z \le 20$	A1	[4]	Correct fully simplified form (cao)
May use slack variables, provided they also specify slack variables non-negative eg $6x + 8y + 5z + t = 50$, $t \ge 0$ = M1, A1			If slack variable form used and fully simplified but without specifying that slack variables are non-negative, SC M1 A0 for each

(ii)

P	x	y	z	s	t	u	RHS
1	-100	-40	-120	0	0	0	0
0	12	20	15	1	0	0	60
0	6	8	5	0	1	0	50
0	3	2	3	0	0	1	20

M1 — Objective row correct <u>and</u> three slack variables used

A1 — Three constraint rows correct (ft (i), if reasonable)
Accept variations in order of rows and columns
Condone P column missing here

(ii)

$60 \div 15 = 4$, $50 \div 5 = 10$, $20 \div 3 = 6\frac{2}{3}$
Pivot on the 15 in the z column

B1 — Correct pivot choice from <u>their z column</u>

New row 2 = row 2 ÷ 15
New row 1 = row 1 + 120 × new row 2
New row 3 = row 3 – 5 × new row 2
New row 4 = row 4 – 3 × new row 2

M1 — Correct method for <u>their</u> pivot row seen (or implied from <u>correct row</u> in tableau if no attempt seen)

A1 — Correct method for their <u>three</u> other rows seen as a <u>formula</u>

P	x	y	z	s	t	u	RHS
1	-4	120	0	8	0	0	480
0	$\frac{4}{5}$	$1\frac{1}{3}$	1	$\frac{1}{15}$	0	0	4
0	2	$1\frac{1}{3}$	0	$-\frac{1}{3}$	1	0	30
0	$\frac{3}{5}$	-2	0	$-\frac{1}{5}$	0	1	8

M1 — Iterate to get a tableau with exactly <u>four basis columns</u> and <u>non-negative entries in final column</u>, in which the value of the <u>objective has not decreased</u>

A1 — Values in final column correct (follow through)

$4 \div \frac{4}{5} = 5$, $30 \div 2 = 15$, $8 \div \frac{3}{5} = 13\frac{1}{3}$
Pivot on the $\frac{4}{5}$ in the x column

B1 — Correct pivot choice for their second iteration

New row 2 = row 2 ÷ $\frac{4}{5}$
New row 1 = row 1 + 4 × new row 2
New row 3 = row 3 – 2 × new row 2
New row 4 = row 4 – $\frac{3}{5}$ × new row 2

M1 — Correct method for <u>their</u> pivot row seen (or implied from <u>correct row</u> in tableau if no attempt seen)

A1 — Correct method for their <u>three</u> other rows seen as a <u>formula</u>

P	x	y	z	s	t	u	RHS
1	0	$126\frac{2}{3}$	5	$8\frac{1}{3}$	0	0	500
0	1	$1\frac{2}{3}$	$1\frac{1}{4}$	$\frac{1}{12}$	0	0	5
0	0	-2	$-2\frac{1}{2}$	$-\frac{1}{2}$	1	0	20
0	0	-3	$-\frac{3}{4}$	$-\frac{1}{4}$	0	1	5

M1 — Iterate to get a tableau with exactly <u>four basis columns</u> and <u>non-negative entries in final column</u>, in which the value of the <u>objective has not decreased</u>

A1 — Values in final column correct (follow through)

	Make 5 litres of *fruit salad* only	B1	[13]	Interpretation of <u>their</u> final (non-negative) <u>x, y and z</u>, in context (need 'only' or equivalent; '5 *fruit salads*' is not enough) $x = 5$, $y = 0$, $z = 0$ gives B0
(iii)	$60 \div 12 = 5$, $50 \div 6 = 8\frac{1}{3}$, $20 \div 3 = 6\frac{2}{3}$ Pivot on the 12 in the x column New row 2 = row 2 ÷ 12 New row 1 = row 1 + 100 × new row 2 Showing that there are no negative entries in objective row Saying that optimum has been achieved ('no negatives in top row')	B1 M1 A1 M1 A1	[5]	Correct pivot choice <u>from their x column</u> Correct method for <u>their</u> pivot row (seen or implied from correct row in tableau) Correct method for their <u>objective</u> row seen as a formula Showing that there are no negative entries in objective row Or achieving a final tableau, in one iteration, with exactly four basis columns and non-negative entries in final column, in which the value of the objective has not decreased