Mark Scheme 4727
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1 Directions $[1, 1, -1]$ and $[2, -3, 1]$	B1	For identifying both directions (may be implied by working)
$\theta = \cos^{-1}\frac{\lvert[1, 1, -1].[2, -3, 1]\rvert}{\sqrt{3}\ \sqrt{14}}$	M1	For using scalar product of their direction vectors
$= \cos^{-1}\frac{\lvert -2\rvert}{\sqrt{42}}$	M1	For completely correct process for their angle
$= 72.0°, 72°$ or 1.26 rad	A1 **4** $\boxed{4}$	For correct answer
2 **(i)** Identities b, 6	B1 B1	For correct identities
Subgroups $\{b, d\}$, $\{6, 4\}$	B1 B1 **4**	For correct subgroups
(ii) $\{a, b, c, d\} \leftrightarrow \{2, 6, 8, 4\}$ or $\{8, 6, 2, 4\}$	B1 B1	For $b \leftrightarrow 6$, $d \leftrightarrow 4$
	B1 **3**	For $a, c \leftrightarrow 2, 8$ in either order
	$\boxed{7}$	**SR** If B0 B0 B0 then M1 A1 may be awarded for stating the orders of all elements in G and H
3 **(i)** $3y^2\frac{dy}{dx} = \frac{dz}{dx}$	M1	For differentiating substitution
$\Rightarrow \frac{dz}{dx} + 2xz = e^{-x^2}$	A1	For resulting equation in z and x
Integrating factor $\left(e^{\int 2x\,dx} =\right)e^{x^2}$	B1 √	For correct IF f.t. for an equation in suitable form
$\Rightarrow \frac{d}{dx}\left(ze^{x^2}\right)$ *OR* $\frac{d}{dx}\left(y^3e^{x^2}\right) = 1$	M1	For using IF correctly
$\Rightarrow z\,e^{x^2}$ *OR* $y^3e^{x^2} = x\ (+c)$	A1	For correct integration (+c not required here)
$\Rightarrow y = (x+c)^{\frac{1}{3}}\ e^{-\frac{1}{3}x^2}$	A1 **6**	For correct answer **AEF**
(ii) As $x \to \infty$, $y \to 0$	B1 **1** $\boxed{7}$	For correct statement
4 **(i)** $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$, $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$	B1	For either expression, seen or implied z may be used for $e^{i\theta}$ throughout
$\Rightarrow \cos^2\theta\sin^4\theta = \frac{1}{4}\left(e^{i\theta} + e^{-i\theta}\right)^2\frac{1}{16}\left(e^{i\theta} - e^{-i\theta}\right)^4$		
$= \frac{1}{4}\left(e^{2i\theta} + 2 + e^{-2i\theta}\right).\frac{1}{16}\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$	M1 A1 A1	For expanding terms For the 2 correct expansions **SR** Allow A1 A0 for $k\left(e^{2i\theta} + 2 + e^{-2i\theta}\right)\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$, $k \neq \frac{1}{64}$
$= \frac{1}{64}\left(\left(e^{6i\theta} + e^{-6i\theta}\right) - 2\left(e^{4i\theta} + e^{-4i\theta}\right) - \left(e^{2i\theta} + e^{-2i\theta}\right) + 4\right)$	M1	For grouping terms and using multiple angles
$= \frac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2)$ **AG**	A1 **6**	For answer obtained correctly

(ii) $\int_0^{\frac{1}{3}\pi} \cos^2\theta \sin^4\theta \, d\theta =$		
$= \frac{1}{32}\left[\frac{1}{6}\sin 6\theta - \frac{1}{2}\sin 4\theta - \frac{1}{2}\sin 2\theta + 2\theta\right]_0^{\frac{1}{3}\pi}$	M1 A1	For integrating answer to **(i)** For all terms correct
$= \frac{1}{32}\left[0 + \frac{1}{4}\sqrt{3} - \frac{1}{4}\sqrt{3} + \frac{2}{3}\pi - 0\right] = \frac{1}{48}\pi$	A1 **3** $\boxed{9}$	For correct answer
5 (i) *EITHER* $z = \sqrt{8}\,\text{cis}(2k+1)\frac{\pi}{4},\ k = 0, 1, 2, 3$	B1	For correct modulus **AEF**
OR $z = \sqrt{8}\,e^{(2k+1)\frac{\pi}{4}i}, k = 0, 1, 2, 3$	B1 **2**	For correct arguments **AEF**
(ii)		
$z = 2\sqrt{2}\left\{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right\}$	B1	For any of $\pm\frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}i$
$z = 2+2i, -2+2i, -2-2i, 2-2i$	B1	For any one value of z correct
	B1	For all values of z correct **AEFcartesian** (may be implied from symmetry or factors)
$(z-\alpha), (z-\beta), (z-\gamma), (z-\delta)$	B1 √ **4**	f.t., where $\alpha, \beta, \gamma, \delta$ are answers above
(iii) *EITHER* $(z-(2+2i))(z-(2-2i))$	M1	For combining factors from **(ii)** in pairs
$\times (z-(-2+2i))(z-(-2-2i))$	M1	Use of complex conjugate pairs
$= (z^2+4z+8)(z^2-4z+8)$	A1	For correct answer
OR $z^4 + 64 = (z^2+az+b)(z^2+cz+d)$	M1	For equating coefficients
$\Rightarrow a+c=0, b+ac+d=0, ad+bc=0, bd=64$	M1	For solving equations
Obtain $(z^2+4z+8)(z^2-4z+8)$	A1 **3** $\boxed{9}$	For correct answer
6 (i) $\mathbf{MB} = [2, 1, -2]$, $\mathbf{OF} = [4, 1, 2]$ $\mathbf{MB} \times \mathbf{OF}$	B1 M1	For either vector correct (allow multiples) For finding vector product of their **MB** and **OF**
$= [4, -12, -2]$ *OR* $k[2, -6, -1]$	A1 **3**	For correct vector
(ii) *EITHER* Find vector product of any two of $\pm[2, -1, 2]$, $\pm[0, 0, 2]$, $\pm[2, -1, 0]$ and any two of $\pm[4, 0, 2]$, $\pm[4, -1, 0]$, $\pm[0, 1, 2]$	M1	For finding two relevant vector products
Obtain $k[1, 2, 0]$	A1	For correct LHS of plane *CMG*
Obtain $k[1, 4, -2]$	A1	For correct LHS of plane *OEG*
	M1	For substituting a point into each equation
$x+2y=2$ and $x+4y-2z=0$	A1	For both equations correct **AEF**
OR Use $ax+by+cz=d$ with coordinates of *C*, *M*, *G* *OR* *O*, *E*, *G* substituted	M1	For use of cartesian equation of plane
Obtain $a:b:c = 1:2:0$ for *CMG*	A1	For correct ratio
Obtain $a:b:c = 1:4:-2$ for *OEG*	A1	For correct ratio
	M1	For substituting a point into each equation
$x+2y=2$ and $x+4y-2z=0$	A1 **5**	For both equations correct **AEF**

(iii) *EITHER* Put x, y *OR* $z = t$ in planes *OR* evaluate $k[1, 2, 0] \times k[1, 4, -2]$ Obtain $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ where $\mathbf{a} = [0, 1, 2], [2, 0, 1]$ *OR* $[4, -1, 0]$ $\mathbf{b} = k[-2, 1, 1]$	M1 A1 A1 **3** $\boxed{11}$	For solving plane equations in terms of a parameter *OR* for finding vector product of normals to planes from **(ii)** Obtain a correct point **AEF** Obtain correct direction **AEF**
7 (i) $(x^{-1}ax)^m = (x^{-1}ax)(x^{-1}ax)\ldots(x^{-1}ax)$ $= x^{-1}aa\ldots ax$, associativity, $xx^{-1} = e$ $= x^{-1}a^m x = x^{-1}ex$ when $m = n$, not $m < n$ $= x^{-1}x$ $= e \Rightarrow$ order n	M1 A1 A1 B1 A1 A1 **6**	For considering powers of $x^{-1}ax$ For using associativity and inverse properties For using order of *a* correctly For using property of identity For correct conclusion
(ii) ***EITHER*** $(x^{-1}ax)z = e$ $\Rightarrow axz = xe = x \Rightarrow xz = a^{-1}x$ $\Rightarrow z = x^{-1}a^{-1}x$	M1 A1 A1	For attempt to solve for *z* **AEF** For using pre- or post multiplication For correct answer
OR Use $(pq)^{-1} = q^{-1}p^{-1}$ *OR* $(pqr)^{-1} = r^{-1}q^{-1}p^{-1}$ State $(x^{-1})^{-1} = x$ Obtain $x^{-1}a^{-1}x$	M1 A1 A1 **3**	For applying inverse of a product of elements For stating this property For correct answer with no incorrect working **SR** correct answer with no working scores B1 only
(iii) $ax = xa \Rightarrow x = a^{-1}xa$ $\Rightarrow xa^{-1} = a^{-1}x$	M1 A1 **2** $\boxed{11}$	Start from commutative property for *ax* Obtain commutative property for $a^{-1}x$
8 (i) $m^2 + 2km + 4 = 0$ $\Rightarrow m = -k \pm \sqrt{k^2 - 4}$ **(a)** $x = \mathrm{e}^{-kt}\left(A\mathrm{e}^{\sqrt{k^2-4}\,t} + B\mathrm{e}^{-\sqrt{k^2-4}\,t}\right)$	M1 A1 **2** M1 A1 **2**	For stating and attempting to solve auxiliary eqn For correct solutions, at any stage **AEF** For using $\mathrm{e}^{\mathrm{f}(t)}$ with distinct real roots of aux eqn For correct answer **AEF**
(b) $x = \mathrm{e}^{-kt}\left(A\mathrm{e}^{\mathrm{i}\sqrt{4-k^2}\,t} + B\mathrm{e}^{-\mathrm{i}\sqrt{4-k^2}\,t}\right)$ $x = \mathrm{e}^{-kt}\left(A'\cos\sqrt{4-k^2}\,t + B'\sin\sqrt{4-k^2}\,t\right)$ *OR* $x = \mathrm{e}^{-kt}\left(C' \begin{matrix}\cos\\ \sin\end{matrix}\left(\sqrt{4-k^2}\,t + \alpha\right)\right)$	M1 A1 **2**	For using $\mathrm{e}^{\mathrm{f}(t)}$ with complex roots of aux eqn This form may not be seen explicitly but if stated as final answer earns M1 A0 For correct answer
(c) $x = \mathrm{e}^{-2t}(A'' + B''t)$	M1 A1 **2**	For using $\mathrm{e}^{\mathrm{f}(t)}$ with equal roots of aux eqn For correct answer. Allow *k* for 2

(ii)(a) $x = B'\mathrm{e}^{-t}\sin\sqrt{3}\,t$	B1 √	For using $t = 0,\ x = 0$ correctly. f.t. from **(b)**
$\dot{x} = B'\mathrm{e}^{-t}\left(\sqrt{3}\cos\sqrt{3}\,t - \sin\sqrt{3}\,t\right)$	M1 A1 √	For differentiating *x* For correct expression. f.t. from their *x*
$t = 0,\ \dot{x} = 6 \Rightarrow B' = 2\sqrt{3}$, $x = 2\sqrt{3}\mathrm{e}^{-t}\sin\sqrt{3}\,t$	A1 **4**	For correct solution **AEF** **SR** √ and **AEF** OK for $x = C'\mathrm{e}^{-t}\cos\left(\sqrt{3}\,t + \frac{1}{2}\pi\right)$
(b) $x \to 0$	B1	For correct statement
$\mathrm{e}^{-t} \to 0$ and sin( ) is bounded	B1 **2** $\boxed{14}$	For both statements