

Version 1.0: 0106



General Certificate of Education

Mathematics 6360

MM2B Mechanics 2

Mark Scheme

2006 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

| | | | |
|--------------|--|-----|----------------------------|
| M | mark is for method | | |
| m or dM | mark is dependent on one or more M marks and is for method | | |
| A | mark is dependent on M or m marks and is for accuracy | | |
| B | mark is independent of M or m marks and is for method and accuracy | | |
| E | mark is for explanation | | |
| ✓ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| -x EE | deduct x marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM2B

| Q | Solution | Marks | Total | Comment |
|----------------|---|-----------------------|----------|--|
| 1(a) | $KE = \frac{1}{2} \times 0.4 \times 8^2 = 12.8 \text{ J}$ | M1 A1 | 2 | Use of KE formula. Correct KE |
| (b)(i) | $KE = 12.8 + 0.4 \times 9.8 \times 6 = 36.32 \text{ J}$ AG | M1 A1 | 2 | Calculation of GPE Correct KE from correct expression (Allow use of CA equations in solutions) |
| (ii) | $\frac{1}{2} \times 0.4v^2 = 36.32$ $v = \sqrt{\frac{36.32 \times 2}{0.4}} = 13.5 \text{ ms}^{-1}$ | M1 A1 A1 | 3 | Two term energy equation Correct energy equation Correct speed |
| (iii) | No air resistance No resistance forces Weight is the only force | B1 | 1 | Appropriate assumption |
| Total | | | 8 | |
| 2(a) | $T \cos 30^\circ = 2 \times 9.8$ $T = \frac{2 \times 9.8}{\cos 30^\circ}$ $T = 22.6 \text{ N}$ AG | M1 A1 A1 | 3 | Resolving vertically with two terms Correct equation Correct T from correct working |
| (b) | $T \cos 60^\circ = 2 \times \frac{v^2}{0.6}$ $v = 1.84 \text{ ms}^{-1}$ | M1 A1 dM1 A1 | 4 | Resolving horizontally. Correct equation Solving for v Correct v |
| Total | | | 7 | |
| 3(a)(i) | $a = 2 + 12e^{-t}$ | M1 A1 | 2 | Differentiating, with at least one term correct. Correct velocity |
| (ii) | $2 < a \leq 14$ | B1 B1 B1 | 3 | For 2 For 14 Correct inequalities |
| (b) | $s = t^2 + 12e^{-t} + c$ $s = 0, t = 0 \Rightarrow c = -12$ $s = t^2 + 12e^{-t} - 12$ | M1 A1 dM1 A1 | 4 | Integrating, with at least one term correct Correct expression with or without c Finding c . Correct final expression |
| Total | | | 9 | |

MM2B cont

| Q | Solution | Marks | Total | Comment |
|--------------|--|-----------------------------|--------------|--|
| 4(a) | Because the lamina is symmetrical. | B1 | 1 | Correct explanation |
| (b) | $\bar{y} = \frac{250 \times 2.5 + 150 \times 7.5}{250 + 150}$ $= \frac{1750}{400}$ $= 4.375 \quad \text{AG}$ | M1 A1 A1 A1 | 4 | Moment equation with appropriate number of terms Correct numerator correct denominator Correct value from correct working |
| (c) | $\tan \alpha = \frac{10 - 4.375}{25} = \frac{5.625}{25}$ $\alpha = 12.7^\circ$ | M1 M1 A1 A1 | 4 | Use of tan. Subtracting from 10 Correct expression Correct angle |
| (d) | When it has been assumed that the centre of mass of each of the rectangles used is at its centre. OR Relating area to mass. | B1 | 1 | Correct explanation |
| Total | | | 10 | |
| 5(a) | $\mathbf{F} = 12 \cos t \mathbf{i} - 30 \sin t \mathbf{j}$ $\mathbf{F}(0) = 6 \times 2 \mathbf{i} \text{ so } F = 12 \text{ N}$ | M1 A1 A1 | 3 | Use of $\mathbf{F} = m\mathbf{a}$ Correct \mathbf{F} Correct magnitude |
| (b) | $\mathbf{v} = \int 2 \cos t dt \mathbf{i} + \int -5 \sin t dt \mathbf{j}$ $= (2 \sin t + c_1) \mathbf{i} + (5 \cos t + c_2) \mathbf{j}$ $\mathbf{v}(0) = 2 \mathbf{i} + 10 \mathbf{j} \Rightarrow c_1 = 2, c_2 = 5$ $\mathbf{v} = (2 \sin t + 2) \mathbf{i} + (5 \cos t + 5) \mathbf{j}$ | M1 A1 A1 dM1 A1 | 5 | Integrating Correct \mathbf{i} component Correct \mathbf{j} component Both of above with or without constants Finding constants of integration Correct final answer |
| Total | | | 8 | |

MM2B (cont)

| Q | Solution | Marks | Total | Comment |
|---------------|---|-----------|-----------|---|
| 6(a) | The boat is not affected by the movement of the water. | B1 | | First assumption |
| | The resistance force will directly oppose the motion of the boat and be the only force that needs to be considered. | B1 | 2 | second assumption |
| (b) | The boat is a particle. / There is no wind. / No air resistance. | B1 | 1 | Appropriate assumption |
| (c)(i) | $80 \frac{dv}{dt} = -20v$ | M1 | | Use of Newton's second law, $\frac{dv}{dt}$ and $20v$. |
| | $\frac{dv}{dt} = -\frac{v}{4}$ | A1 | 2 | Correct result |
| (ii) | $\int \frac{1}{v} dv = -\int \frac{1}{4} dt$ | M1 | | Sep. of variables and forming integrals |
| | $\ln v = -\frac{t}{4} + c$ | dM1 A1 | | Integrating to get an $\ln v$ term Correct integrals with or without c |
| | $v = Ae^{-\frac{t}{4}}$ $v = 12, t = 0 \Rightarrow A = 12$ | dM1 | | Finding A or c |
| | $v = 12e^{-\frac{t}{4}}$ | A1 | 5 | Correct final result |
| Total | | | 10 | |
| 7(a) | $\frac{1}{2}mv^2 = \frac{1}{2}m \times 2^2 + mg(3 - 3 \cos \theta)$ | M1 A1 | | Three term energy equation. Correct equation |
| | $v^2 = 4 + 6g(1 - \cos \theta)$ | dM1 A1 | 4 | Solving for v^2 Correct result from correct working |
| (b) | $mg \cos \theta = m \frac{v^2}{3}$ | M1 A1 | | Resolving towards the centre Correct equation |
| | $3g \cos \theta = 4 + 6g - 6g \cos \theta$ | dM1 | | Solving for $\cos \theta$ |
| | $\cos \theta = \frac{4 + 6g}{9g}$ $\theta = 44.6^\circ$ | A1 A1 | 5 | Correct $\cos \theta$ Correct angle |
| Total | | | 9 | |

MM2B (cont)

| Q | Solution | Marks | Total | Comment |
|---------------|---|-------|----------------------------|---|
| 8(a) | $\frac{100}{0.4}e = 10 \times 9.8$ | M1 | | Use of Hookes law and equilibrium |
| | $e = 0.392 \text{ m}$ Length = $0.392 + 0.4 = 0.792$ | A1 | 2 | Correct length |
| (b) | $EPE = \frac{1}{2} \times \frac{100}{0.4} \times 0.6^2 = 45 \text{ J}$ AG | M1 | | Use of EPE formula |
| | | A1 | 2 | Correct value from correct working |
| (c)(i) | $45 = \frac{1}{2} \times \frac{100}{0.4} (x - 0.4)^2 + \frac{1}{2} \times 10v^2 + 10 \times 9.8(1 - x)$ | M1 | | Expression for EPE with $(x \pm 0.4)^2$ |
| | | A1 | | Correct EPE |
| | $45 = 125(x - 0.4)^2 + 5v^2 + 98(1 - x)$ | M1 | | Four term energy equation |
| | $5v^2 = 98x - 98 + 45 - 125x^2 + 100x - 20$ | B1 | | Correct GPE |
| | $v^2 = 39.6x - 25x^2 - 14.6$ | A1 | | Correct equation |
| | | dM1 | | Solving for v^2 |
| | AG | A1 | 7 | Correct result from correct working |
| (ii) | $39.6x - 25x^2 - 14.6 = 0$ | | | |
| | $25x^2 - 39.6x + 14.6 = 0$ | M1 | | Solving quadratic |
| | $x = \frac{39.6 \pm \sqrt{39.6^2 - 4 \times 25 \times 14.6}}{2 \times 25}$ | | | |
| | $= 1 \text{ or } 0.584$ $x = 0.584$ | A1 | | Correct solutions |
| | A1 | 3 | Appropriate value selected | |
| | | | | SC Only correct answers given award M1A1. |
| | Total | | 14 | |
| | TOTAL | | 75 | |